COMPLEMENTOS DE OPÇÕES (MSc FINANÇAS 2005/2006)
OPÇÕES EXÓTICAS (MSc MAT. FINANCEIRA 2005/2006)
EXAME - Resolução

21/07/06 **Duração: 2.5 horas**

<u>CASO 1</u>

a) O payoff terminal (no momento "T") de uma *European and asymmetric power call* sobre o activo "S" e com *strike* "X" é definido como sendo igual a: $\left[(S_T)^n - X\right]^+$, para $n \in \Re \setminus \{0\}$. Pretende-se que formule a avaliação de tal contrato no momento "t" ($\leq T$), assumindo os pressupostos do modelo de Merton.

$$APC_t(S, X, T; n) = e^{-r(T-t)} \times E_Q\left\{\left[(S_T)^n - X\right]^+ \middle| F_t\right\}.$$

Assumindo que "S" segue um GBM,

$$S_T = S_t \exp\left\{\left[r - q - \frac{\sigma^2}{2}\right](T-t) + \sigma\int_t^T dW_u\right\}$$

$$\Downarrow$$

$$(S_T)^n = (S_t)^n \exp\left[n\left(r - q - \frac{\sigma^2}{2}\right)(T-t) + n\sigma\int_t^T dW_u\right]$$

Consequentemente,

$$APC_t(S, X, T; n) = e^{-r(T-t)} \times E_Q\left\{\left[(S_t)^n \exp\left[n\left(r - q - \frac{\sigma^2}{2}\right)(T-t) + n\sigma\int_t^T dW_u\right] - X\right]^+ \middle| F_t\right\}$$

$$= e^{-r(T-t)} \times \int_{z^*}^{\infty}\left\{(S_t)^n \exp\left[n\left(r - q - \frac{\sigma^2}{2}\right)(T-t) + n\sigma z\right] - X\right\}\frac{1}{\sqrt{2\pi(T-t)}}\exp\left(-\frac{1}{2}\frac{z^2}{T-t}\right)dz,$$

onde

$$z^* = \frac{\ln\left[\frac{X}{(S_t)^n}\right] - n\left(r - q - \frac{\sigma^2}{2}\right)(T-t)}{n\sigma} = \frac{\ln\left[\frac{X^n}{(S_t)^n}\right] + \ln\left(\frac{X}{X^n}\right) - n\left(r - q - \frac{\sigma^2}{2}\right)(T-t)}{n\sigma}$$

$$= \frac{\ln\left(\frac{X}{S_t}\right) - \left(r - q - \frac{\sigma^2}{2}\right)(T-t)}{\sigma} - \frac{n-1}{n\sigma}\ln(X).$$

I.e.

$APC_t(S, X, T; n)$

$$= (S_t)^n \exp\left[\left((n-1)r - nq - n\frac{\sigma^2}{2}\right)(T-t)\right]\int_{z*}^{\infty}\frac{1}{\sqrt{2\pi(T-t)}}\exp\left[-\frac{1}{2}\frac{z^2 - 2(T-t)n\sigma z}{T-t}\right]dz$$

$$-e^{-r(T-t)}X\int_{z*}^{\infty}\frac{1}{\sqrt{2\pi(T-t)}}\exp\left(-\frac{1}{2}\frac{z^2}{T-t}\right)dz$$

$$= (S_t)^n \exp\left[\left((n-1)r - nq - n\frac{\sigma^2}{2}\right)(T-t) + \frac{(T-t)^2 n^2\sigma^2}{2(T-t)}\right]\int_{z*}^{\infty}\frac{1}{\sqrt{2\pi(T-t)}}\exp\left\{-\frac{1}{2}\frac{\left[z^2 - (T-t)n\sigma\right]^2}{T-t}\right\}dz$$

$$-e^{-r(T-t)}X\Phi\left(-\frac{z*}{\sqrt{T-t}}\right)$$

$$= (S_t)^n \exp\left[\left((n-1)r - nq + (n^2 - n)\frac{\sigma^2}{2}\right)(T-t)\right]\Phi\left(-\frac{z* - (T-t)n\sigma}{\sqrt{T-t}}\right)$$

$$-e^{-r(T-t)}X\Phi\left(\frac{\ln\left[\frac{(S_t)^n}{X}\right] + n\left(r - q - \frac{\sigma^2}{2}\right)(T-t)}{n\sigma\sqrt{T-t}}\right)$$

$$= (S_t)^n \exp\left[\left((n-1)r - nq + (n^2 - n)\frac{\sigma^2}{2}\right)(T-t)\right]\Phi\left(\frac{\ln\left[\frac{(S_t)^n}{X}\right] + n\left(r - q - \frac{\sigma^2}{2}\right)(T-t)}{n\sigma\sqrt{T-t}} + \frac{(T-t)n\sigma}{\sqrt{T-t}}\right)$$

$$-e^{-r(T-t)}X\Phi\left(\frac{\ln\left[\frac{(S_t)^n}{X}\right] + n\left(r - q - \frac{\sigma^2}{2}\right)(T-t)}{n\sigma\sqrt{T-t}}\right).$$

I.e.

$$APC_t\left(S,X,T;n\right)$$

$$=\left(S_t\right)^n \exp\left[\left(\left(n-1\right)r-nq+\left(n^2-n\right)\frac{\sigma^2}{2}\right)\left(T-t\right)\right]\Phi\left(d_p+\sqrt{T-t}\,n\sigma\right)-e^{-r\left(T-t\right)}X\Phi\left(d_p\right).$$

sendo

$$d_p=\frac{\ln\left[\frac{\left(S_t\right)^n}{X}\right]+n\left(r-q-\frac{\sigma^2}{2}\right)\left(T-t\right)}{n\sigma\sqrt{T-t}}=\frac{\ln\left[\frac{\left(S_t\right)^n}{X^n}\right]+\ln\left(\frac{X^n}{X}\right)+n\left(r-q-\frac{\sigma^2}{2}\right)\left(T-t\right)}{n\sigma\sqrt{T-t}}$$

$$=\frac{\ln\left(\frac{S_t}{X}\right)+\left(r-q-\frac{\sigma^2}{2}\right)\left(T-t\right)}{\sigma\sqrt{T-t}}+\frac{n-1}{n\sigma\sqrt{T-t}}\ln\left(X\right).$$

b) Considere um contrato que paga ao seu titular no vencimento (momento "T") uma cash flow igual a "M" euros caso a cotação do índice DAX não desça abaixo do valor $He^{h\left(u-t\right)}, H>0$, em qualquer momento $u\in\left[t,T\right]$. Pretende-se que avalie tal contrato no momento "t" ($\leq T$).

Valor terminal do contrato:

$$V_T=M\times 1_{\left\{\inf\limits_{t\leq u\leq T}\left(S_u\right)>He^{h\left(u-t\right)}\right\}}.$$

Valor actual do contrato:

$$V_t=e^{-r\left(T-t\right)}\times M\times E_Q\left(1_{\left\{\inf\limits_{t\leq u\leq T}\left(S_u\right)>He^{h\left(u-t\right)}\right\}}\middle|F_t\right)$$

$$\Updownarrow$$

$$V_t=e^{-r\left(T-t\right)}\times M\times Q\left[\inf_{t\leq u\leq T}\left(S_u\right)>He^{h\left(u-t\right)}\middle|F_t\right]$$

$$\Updownarrow$$

$$V_t=e^{-r\left(T-t\right)}\times M\times Q\left[\inf_{t\leq u\leq T}\left(S_u e^{-h\left(u-t\right)}\right)>H\middle|F_t\right]$$

$$\Updownarrow$$

$$V_t=e^{-r\left(T-t\right)}\times M\times Q\left[\inf_{t\leq u\leq T}\left(\overline{y}_u\right)>\ln\left(\frac{H}{S_t}\right)\middle|F_t\right],(*)$$

onde $\overline{y}_u=\ln\left[\frac{S_u}{S_t}e^{-h\left(u-t\right)}\right]$.

Uma vez que

$$y_u = \ln\left(\frac{S_u}{S_t}\right) = \left(r - q - \frac{\sigma^2}{2}\right)(u-t) + \sigma\int_t^u dW_v,$$

então

$$\overline{y}_u = \ln\left[\frac{S_u}{S_t}e^{-h(u-t)}\right] = \left[r - (q+h) - \frac{\sigma^2}{2}\right](u-t) + \sigma\int_t^u dW_v$$

segue também um aritmetic BM.

Portanto, comparando a equação (*) com a equação (124), o fair value do contrato pode ser obtido via equação (122) com $\eta = -1$ e substituindo $\mu := r - q - \frac{\sigma^2}{2}$ por $\overline{\mu} := r - (q+h) - \frac{\sigma^2}{2}$. Assim,

$$V_t = e^{-r(T-t)} \times M \times \left\{ \Phi\left[d_2^M(S_t, H)\right] - \left(\frac{H}{S_t}\right)^{\frac{2\overline{\mu}}{\sigma^2}} \Phi\left[d_2^M(H, S_t)\right] \right\}.$$

a) Defina uma carteira de opções capaz de gerar daqui a 1 ano a diferença entre as cotações máxima e mínima registadas pela acção GN durante o próximo ano.

Carteira de *floating strike lookback options* a constituir (assumindo monitorização contínua do spot):

	Payoff hoje	Payoff daqui a 1 ano
Long lookback call	$-c_0^L$	$S_1 - S_{\min}$
Long lookback put	$-p_0^L$	$S_{\max} - S_1$
Total:	$-c_0^L - p_0^L$	$S_{\max} - S_{\min}$

CASO 2

a)

Taxa spot a 6M com capitalização continua:

$$r: \quad e^{r \times 6/12} = 1 + 3\% \times 6/12 \Rightarrow r = \frac{12}{6}\ln\left(1 + 3\% \times 6/12\right) \cong 2.978\%.$$

$$B_0 = 100\% \times e^{-2.978\% \times 0.5} + RV_0.$$

$$RV_{6M} = \begin{cases} 5\% \Leftarrow S_{6M} > 3{,}425 \times 1.1 = 3{,}767.50 \\ 0\% \Leftarrow ELSE \end{cases}$$

$$= 5\% \times D(1)_{6M}\left(M = 1; S; X = 3{,}767.50; T = 6M\right).$$

Portanto,

$$\begin{aligned} RV_0 &= 5\% \times D(1)_{6M}\left(M = 1; S; X = 3{,}767.50; T = 6M\right) \\ &= 5\% \times e^{-2.978\% \times 0.5} \times \Phi\left[d_2^M\left(3{,}767.50\right)\right]. \end{aligned}$$

$$\Phi\left[d_2^M\left(3{,}767.50\right)\right] = \Phi\left[\frac{\ln\left(3{,}425 / 3{,}767.50\right) + \left(2.978\% - 3.58\% - (0.1663)^2 / 2\right) \times 0.5}{0.1663 \times \sqrt{0.5}}\right]$$

$$\cong \Phi(-0.8949) = 0.1854.$$

$$RV_0 = 5\% \times e^{-2.978\% \times 0.5} \times 0.1854 = 0.91\%.$$

$B_0 = 98.52\% + 0.91\% \cong 99.43\% < 100\% \Rightarrow$ Não investir.

b)

$$B_0 = 100\% \times e^{-2.978\% \times 0.5} + RV_0.$$

$$RV_{6M} = 50\% \times \begin{cases} \dfrac{S_{3M} - S_{6M}}{S_{3M}} \Leftarrow S_{6M} < S_{3M} \\ 0\% \Leftarrow ELSE \end{cases}$$

$$= \frac{50\%}{E_{3M}} \times \begin{cases} S_{3M} - S_{6M} \Leftarrow S_{6M} < S_{3M} \\ 0 \Leftarrow ELSE \end{cases}$$

$$= \frac{50\%}{S_{3M}} \times p_{6M}\left(S_{6M}; X = E_{3M}; T = 6M\right)$$

$$RV_{3M} = \frac{50\%}{E_{3M}} \times e^{-r\times 0.25} \times E_Q\left[p_{6M}\left(S_{3M}; X = E_{3M}; T = 6M\right) \middle| F_{3M}\right]$$

$$= \frac{50\%}{E_{3M}} \times p_{3M}\left(S_{3M}; X = E_{3M}; T = 6M\right)$$

$$= 50\% \times p_{3M}\left(1; X = 1; T = 6M\right)$$

Consequentemente,

$$RV_0 = \frac{50\% \times p_{3M}\left(1; X = 1; T = 6M\right)}{1 + 2.989\% \times \frac{3}{12}}.$$

A anterior *put* pode ser avaliada via modelo de Merton:

Taxa spot a 3M com capitalização continua:

$$r: \quad e^{r\times 3/12} = 1 + 2.989\% \times 3/12 \Rightarrow r = \frac{12}{3}\ln\left(1 + 2.989\% \times 3/12\right) \cong 2.978\%.$$

$$p_{3M}\left(1; X = 1; T = 6M\right) = -e^{-3.58\%\times 0.25} \times \Phi\left(-d_1^M\right) + e^{-2.978\%\times 0.25} \times \Phi\left(-d_2^M\right).$$

$$\Phi\left(-d_1^M\right) = \Phi\left[-\frac{\ln\left(1/1\right) + \left(2.978\% - 3.58\% + \frac{(0.1663)^2}{2}\right)\times 0.25}{0.1663 \times \sqrt{0.25}}\right]$$
$$= \Phi\left(-0.0235\right) = 1 - \Phi\left(0.0235\right) = 1 - 0.5094 = 0.4906.$$

$$\Phi\left(-d_2^M\right) = \Phi\left(-0.0235 + 0.1663 \times \sqrt{0.25}\right)$$
$$= N\left(0.0597\right) \cong 0.5238.$$

Portanto,

$$p_{3M}\left(1; X = 1; T = 6M\right) = -e^{-3.58\%\times 0.25} \times 0.4906 + e^{-2.978\%\times 0.25} \times 0.5238 \cong 0.0336.$$

Em alternativa, visto que as taxas spot a 3M e a 6M (em regime de capitalização continua) são iguais e como o preço de uma put Europeia standard é uma função homogénea de grau 1 no spot e no strike:

$$p_{3M}(1; X = 1; T = 6M) = \frac{115.23}{3{,}425.00} \cong 0.0336.$$

$$RV_0 = \frac{50\% \times 0.0336}{1 + 2.989\% \times \frac{3}{12}} \cong 1.67\%.$$

$$B_0 = 98.52\% + 1.67\% = 100.19\%.$$

Margem de intermediação = 101.00% - 100.19% = 0.81%.

c)

$$B_0 = 100\% \times e^{-2.978\% \times 0.5} + RV_0.$$

Assumindo taxas de juro constantes para os próximos 6 meses,

$$RV_0 = E_Q\left(5\% \times e^{-2.978\% \times v} \times 1_{\{S_v = 3{,}767.50 \wedge \sup_{0 \le u < v}(S_u)\} < 3{,}767.50 \wedge v \in [0,6M]} \middle| F_0\right).$$

Trata-se portanto do present value de um knock-out non-deferrable rebate igual a 5% e associado a uma up barrier igual a $3{,}425 \times 1.1 = 3{,}767.50$. Utilizando a equação (131) com $\eta = 1$:

$$\mu = 2.978\% - 3.58\% - \frac{(0.1663)^2}{2} \cong \text{-}0.01985.$$

$$\psi = \sqrt{(-0.01985)^2 + 2 \times (0.1663) \times 2.978\%^2} \cong 0.0451782.$$

$$RV_0 = 5\% \times \left\{ \left(\frac{3{,}767.50}{3425.00}\right)^{\frac{-0.01985-0.0451782}{(0.1663)^2}} \times \Phi\left[-\frac{\ln\left(\frac{3{,}767.50}{3425.00}\right) - 0.0451782 \times 0.5}{0.1663 \times \sqrt{0.5}}\right]\right.$$

$$\left. + \left(\frac{3{,}767.50}{3425.00}\right)^{\frac{-0.01985+0.0451782}{(0.1663)^2}} \times \Phi\left[-\frac{\ln\left(\frac{3{,}767.50}{3425.00}\right) + 0.0451782 \times 0.5}{0.1663 \times \sqrt{0.5}}\right]\right\}$$

$$RV_0 = 5\% \times \left\{ \left(\frac{3,767.50}{3425.00} \right)^{\frac{-0.01985-0.0451782}{(0.1663)^2}} \times \Phi[-0.618419] + \left(\frac{3,767.50}{3425.00} \right)^{\frac{-0.01985+0.0451782}{(0.1663)^2}} \times \Phi[-1.002614] \right\}$$

$$= 5\% \times \left\{ \left(\frac{3,767.50}{3425.00} \right)^{\frac{-0.01985-0.0451782}{(0.1663)^2}} \times 0.268149 + \left(\frac{3,767.50}{3425.00} \right)^{\frac{-0.01985+0.0451782}{(0.1663)^2}} \times 0.158024 \right\}$$

$$\cong 5\% \times 0.386749$$

$$\cong 0.019337.$$

Portanto,

$B_0 = 98.52\% + 1.93\% \cong 100.45\% > 100\% \Rightarrow$ Investir.

d)

Valor terminal do depósito:

$$V_{6M} = 100\% \times \begin{cases} e^{r \times 0.5} \Leftarrow v \notin [0,0.5] \\ e^{-r \times v} \times \frac{S_{6M}}{S_v} \Leftarrow v \in [0,0.5] \end{cases}$$

$\Updownarrow$

$$V_{6M} = e^{r \times 0.5} \times 1_{\{v \notin [0,0.5]\}} + e^{-r \times v} \times \frac{S_{6M}}{3,767.50} \times 1_{\{v \in [0,0.5]\}}$$

Valor actual do depósito:

$$V_0 = e^{-r \times 0.5} \times E_Q\left(e^{r \times 0.5} \times 1_{\{v \notin [0,0.5]\}} \middle| F_0 \right) + e^{-r \times 0.5} \times E_Q\left(e^{-r \times v} \times \frac{S_{6M}}{3,767.50} \times 1_{\{v \in [0,0.5]\}} \middle| F_0 \right)$$

$\Updownarrow$

$$V_0 = Q\left(v > 0.5 \middle| F_0\right) + \frac{e^{-r \times 0.5}}{3,767.50} \times E_Q\left(e^{-r \times v} \times S_{6M} \times 1_{\{v \in [0,0.5]\}} \middle| F_0 \right) (*)$$

A probabilidade $Q\left(v > 0.5 \middle| F_0\right)$ pode ser facilmente calculada via equação (130) com $\eta = 1$:

$$Q(v > 0.5|F_0) = \Phi\left[\frac{\ln\left(\frac{3,767.50}{3425.00}\right) + 0.01985 \times 0.5}{0.1663 \times \sqrt{0.5}}\right]$$

$$- \exp\left[\frac{2 \times (-0.01985)}{(0.1663)^2} \ln\left(\frac{3,767.50}{3425.00}\right)\right] \times \Phi\left[\frac{-\ln\left(\frac{3,767.50}{3425.00}\right) + 0.01985 \times 0.5}{0.1663 \times \sqrt{0.5}}\right]$$

$$\cong \Phi(0.8949) - 0.8721 \times \Phi(-0.7261)$$

$$= 0.8133 - 0.8721 \times (1 - 0.7673) \cong 61.04\%.$$

Para calcular o valor esperado $E_Q\left(e^{-r \times v} \times S_{6M} \times 1_{\{v \in [0,0.5]\}} \middle| F_0\right)$, é preferível proceder a uma alteração da medida de probabilidade, tomando como numerário o spot acumulado de dividendos:

$$\frac{e^{-r \times 0.5}}{3,767.50} \times E_Q\left(e^{-r \times v} \times S_{6M} \times 1_{\{v \in [0,0.5]\}} \middle| F_0\right) = \frac{S_0 e^{-q \times 0.5}}{3,767.50} \times E_{Q_S}\left(e^{-r \times v} \times 1_{\{v \in [0,0.5]\}} \middle| F_0\right), (**)$$

onde Q_S é a nova medida de probabilidade associada ao novo numerário $S_t e^{qt}$ e definida através da seguinte Rádon-Nikodym derivative:

$$\left.\frac{dQ_S}{dQ}\right|F_t = \frac{S_T e^{qT}}{S_t e^{qt}} \frac{e^{rt}}{e^{rT}}.$$

Assumindo que "S" segue um GBM, i.e. que

$$S_T = S_t \exp\left\{\left[r - q - \frac{\sigma^2}{2}\right](T - t) + \sigma \int_t^T dW_u^Q\right\},$$

então:

$$\left.\frac{dQ_S}{dQ}\right|F_t = \exp\left[-\frac{1}{2}\int_t^T (-\sigma)^2 du - \int_t^T (-\sigma) dW_u^Q\right].$$

Aplicando o teorema de Girsanov,

$$dW_t^{Q_S} = -\sigma dt + dW_t^Q.$$

Consequentemente,

$$S_T = S_t \exp\left\{\left[r - q - \frac{\sigma^2}{2}\right](T-t) + \sigma\int_t^T \left(\sigma du + dW_u^{Q_S}\right)\right\}$$

$$\Updownarrow$$

$$S_T = S_t \exp\left\{\left[r - q + \frac{\sigma^2}{2}\right](T-t) + \sigma\int_t^T dW_u^{Q_S}\right\}.$$

Comparando as 2 últimas equações constata-se que a função densidade de probabilidade do first passage time do índice pode ser obtida, na nova medida Q_S, via equação (128) com $\eta = 1$ e substituindo $\mu := r - q - \frac{\sigma^2}{2}$ por $\bar{\mu} := r - q + \frac{\sigma^2}{2}$. Note-se ainda que o termo $E_{Q_S}\left(e^{-r\times v} \times 1_{\{v\in[0,0.5]\}} \middle| F_0\right)$ é similar ao lado direito da equação (134) e, portanto, pode ser obtido via equação (131) com $\eta = 1$ e substituindo $\mu := r - q - \frac{\sigma^2}{2}$ por $\bar{\mu} := r - q + \frac{\sigma^2}{2}$:

$$\bar{\mu} = 2.978\% - 3.58\% + \frac{(0.1663)^2}{2} \cong 0.007805$$

$$\psi = \sqrt{0.007805^2 + 2\times(0.1663)\times 2.978\%^2} \cong 0.041327213.$$

$$\frac{S_0 e^{-q\times 0.5}}{3{,}767.50} \times E_{Q_S}\left(e^{-r\times v} \times 1_{\{v\in[0,0.5]\}} \middle| F_0\right)$$

$$= \frac{3{,}425.00 \times e^{-3.58\%\times 0.5}}{3{,}767.50} \times \left\{\left(\frac{3{,}767.50}{3{,}425.00}\right)^{\frac{0.007805\text{-}0.041327213}{(0.1663)^2}} \times \Phi\left[-\frac{\ln\left(\frac{3{,}767.50}{3425.00}\right) - 0.041327213\times 0.5}{0.1663\times\sqrt{0.5}}\right]\right.$$

$$\left. + \left(\frac{3{,}767.50}{3{,}425.00}\right)^{\frac{0.007805+0.041327213}{(0.1663)^2}} \times \Phi\left[-\frac{\ln\left(\frac{3{,}767.50}{3425.00}\right) + 0.041327213\times 0.5}{0.1663\times\sqrt{0.5}}\right]\right\}$$

$$\frac{S_0 e^{-q\times 0.5}}{3,767.50}\times E_{Q_S}\left(e^{-r\times v}\times 1_{\{v\in[0,0.5]\}}\middle|F_0\right)$$

$$=89.30\%\times\left\{\left(\frac{3,767.50}{3,425.00}\right)^{\frac{0.007805-0.041327213}{(0.1663)^2}}\times\Phi[-0.634794]+\left(\frac{3,767.50}{3,425.00}\right)^{\frac{0.007805+0.041327213}{(0.1663)^2}}\times\Phi[-0.98624]\right\}$$

$$=89.30\%\times\left\{\left(\frac{3,767.50}{3,425.00}\right)^{\frac{0.007805-0.041327213}{(0.1663)^2}}\times 0.262781+\left(\frac{3,767.50}{3,425.00}\right)^{\frac{0.007805+0.041327213}{(0.1663)^2}}\times 0.162008\right\}$$

$$\cong 89.30\%\times 0.426010$$

$$\cong 38.04\%.$$

Portanto,

$V_0 = 61.04\% + 38.04\% \cong 99.08\% < 100\% \Rightarrow$ Não investir.

CASO 3

a)

$$r:\quad e^{r\times 6/12} = 1+3\%\times 6/12 \Rightarrow r = \frac{12}{6}\ln\left(1+3\%\times 6/12\right)\cong 2.978\%.$$

$$S_{10,2} = 3,196.67\times\exp\left\{\left[2.978\% - 3.58\% - \frac{(0.1663)^2}{2}\right]\times\frac{1}{12} + 0.1008\times 0.1663\times\sqrt{\frac{1}{12}}\right\}$$

$\cong$ 3,206.87 .

$$\min_{i=1,\ldots,6}\left(S_{2,i}\right) = 3,521.54 > 3,300 \Rightarrow \quad V_{2,6} = \max(\ 4,198.88\ - 3,425.00\ ;0) = 773.88\ .$$

$$\min_{i=1,\ldots,6}\left(S_{10,i}\right) = 3,196.67 < 3,300\ \Rightarrow \quad V_{10,6} = 0.00\ .$$

b)

$$\hat{V}_0 = e^{-2.978\%\times 0.5} \times \frac{773.88+540.96}{10} \cong 129.54 \ .$$

$$\sigma\left(\hat{V}_0\right) = \frac{e^{-2.978\%\times 0.5}}{\sqrt{10}} \times \sqrt{\frac{(773.88)^2 + (540.96)^2 - (773.88+540.96)^2 / 10}{10-1}} \cong 88.04 \ .$$

c)

Via equação (110),

$$DO(1)_0\left(S; X = S_0 = 3{,}425; L = 3{,}300; R = 0; T = 6M\right)$$

$$= c_0\left(S = 3{,}425; X = 3{,}425; T = 6M\right) - \left(\frac{3{,}300}{3{,}425}\right)^{\frac{2\mu}{(0.1663)^2}} \times c_0\left(S = \frac{(3{,}300)^2}{3{,}425}; X = 3{,}425; T = 6M\right).$$

Visto que:

$$\mu = 2.978\% - 3.58\% - \frac{(0.1663)^2}{2} \cong \text{-}0.01985\ ;$$

$$c_0\left(S = 3{,}425; X = 3{,}425; T = 6M\right) = 152.95 \ \ ;$$

e que uma opção financeira é uma função homogénea de grau 1 no *spot* e no *strike*,

$$c_0\left(S = \frac{(3{,}300)^2}{3{,}425}; X = 3{,}425; T = 6M\right)$$

$$= \frac{(3{,}300)^2}{(3{,}425)^2} \times p_0\left(S = 3{,}425; X = \frac{(3{,}425)^3}{(3{,}300)^2} \cong 3{,}689.38; T = 6M\right)$$

$$= \frac{(3{,}300)^2}{(3{,}425)^2} \times 62.99 \cong 58.48.$$

Então:

$$DO(1)_0\left(S; X = S_0 = 3{,}425; L = 3{,}300; R = 0; T = 6M\right)$$

$$= 152.95 - \left(\frac{3{,}300}{3{,}425}\right)^{\frac{2\times(-0.01985)}{(0.1663)^2}} \times 58.48$$

$$\cong 91.26.$$