MSc MATEMÁTICA FINANCEIRA 2007/2008
OPÇÕES EXÓTICAS
EXAME - Resolução

25/07/08 **Duração: 2.5 horas**

<u>CASO 1</u>

a) Em que circunstâncias podemos avaliar uma *up-and-in call* como sendo uma *standard call*?

Desde que a barreira seja inferior ao strike e o rebate seja nulo.

b) Deduza a fórmula de avaliação de uma *floating strike lookback call* posteriormente à data de inicio de vigência do contrato.

Atendendo à equação (148) dos apontamentos, com $\theta = -1$ e $t_0 \leq t \leq T$:

$$L(-1)_t\left(S_t;\inf_{t_0\leq u\leq T}(S_u);T\right)$$

$$= e^{-r\tau}E_Q\left[S_T - \inf_{t_0\leq u\leq T}(S_u)\middle|F_t\right]$$

$$= e^{-r\tau}E_Q\left[S_T|F_t\right] - e^{-r\tau}E_Q\left[\inf_{t_0\leq u\leq T}(S_u)\middle|F_t\right]$$

$$= e^{-r\tau}S_t e^{(r-q)\tau} - e^{-r\tau}E_Q\left\{\min\left[\inf_{t_0\leq u\leq t}(S_u);\inf_{t\leq u\leq T}(S_u)\right]\middle|F_t\right\}$$

$$= S_t e^{-q\tau} - e^{-r\tau}E_Q\left[\inf_{t_0\leq u\leq t}(S_u)\times 1_{\left\{\inf_{t\leq u\leq T}(S_u)>\inf_{t_0\leq u\leq t}(S_u)\right\}}\middle|F_t\right]$$

$$- e^{-r\tau}E_Q\left[\inf_{t\leq u\leq T}(S_u)\times 1_{\left\{\inf_{t\leq u\leq T}(S_u)\leq\inf_{t_0\leq u\leq t}(S_u)\right\}}\middle|F_t\right].$$

Utilizando a última equação da página 57 dos apontamentos e designando $\inf_{t_0\leq u\leq t}(S_u)$ por $m_{t_0}^t$:

$$E_Q\left[\inf_{t_0\le u\le t}(S_u)\times 1_{\left\{\inf_{t\le u\le T}(S_u)>\inf_{t_0\le u\le t}(S_u)\right\}}\middle|F_t\right]$$

$$=\inf_{t_0\le u\le t}(S_u)\times Q\left[\inf_{t\le u\le T}(S_u)>\inf_{t_0\le u\le t}(S_u)\middle|F_t\right]$$

$$=\inf_{t_0\le u\le t}(S_u)\times\left\{\Phi\left[\frac{-m_{t_0}^t+\mu\tau}{\sigma\sqrt{\tau}}\right]-\exp\left(\frac{2\mu m_{t_0}^t}{\sigma^2}\right)\Phi\left[\frac{m_{t_0}^t+\mu\tau}{\sigma\sqrt{\tau}}\right]\right\}.$$

Via equação (154) dos apontamentos:

$$E_Q\left[\inf_{t\le u\le T}(S_u)\times 1_{\left\{\inf_{t\le u\le T}(S_u)\le\inf_{t_0\le u\le t}(S_u)\right\}}\middle|F_t\right]$$

$$=E_Q\left[S_t\exp\left[\inf_{t\le u\le T}\ln\left(\frac{S_u}{S_t}\right)\right]\times 1_{\left\{S_t\exp\left[\inf_{t\le u\le T}\ln\left(\frac{S_u}{S_t}\right)\right]\le m_{t_0}^t\right\}}\middle|F_t\right]$$

$$=S_t\int_{-\infty}^{m_{t_0}^t}e^y\left\{\phi\left[y;\mu\tau,\sigma\sqrt{\tau}\right]+\frac{2\mu}{\sigma^2}\exp\left(\frac{2\mu y}{\sigma^2}\right)\Phi\left[\frac{y+\mu\tau}{\sigma\sqrt{\tau}}\right]\right.$$
$$\left.+\exp\left(\frac{2\mu y}{\sigma^2}\right)\phi\left[y;-\mu\tau,\sigma\sqrt{\tau}\right]\right\}dy.$$

Aplicando integração por partes:

$$E_Q\left[\inf_{t\le u\le T}\left(S_u\right)\times 1_{\left\{\inf\limits_{t\le u\le T}\left(S_u\right)\le \inf\limits_{t_0\le u\le t}\left(S_u\right)\right\}}\middle| F_t\right]$$

$$= S_t\int_{-\infty}^{m_{t_0}^t} e^y\phi\left[y;\mu\tau,\sigma\sqrt{\tau}\right]dy$$
$$+\frac{2\mu}{\sigma^2}S_t\int_{-\infty}^{m_{t_0}^t}\exp\left(y+\frac{2\mu y}{\sigma^2}\right)\Phi\left[\frac{y+\mu\tau}{\sigma\sqrt{\tau}}\right]dy$$
$$+S_t\int_{-\infty}^{m_{t_0}^t}\frac{e^y}{\sqrt{2\pi\sigma^2\tau}}\exp\left[-\frac{(y+\mu\tau)^2-4\mu y\tau}{2\sigma^2\tau}\right]dy$$

$$= S_t\int_{-\infty}^{m_{t_0}^t} e^y\phi\left[y;\mu\tau,\sigma\sqrt{\tau}\right]dy$$
$$+\frac{2\mu}{\sigma^2}S_t\left(1+\frac{2\mu}{\sigma^2}\right)^{-1}\left[\exp\left(y+\frac{2\mu y}{\sigma^2}\right)\Phi\left[\frac{y+\mu\tau}{\sigma\sqrt{\tau}}\right]\right]_{-\infty}^{m_{t_0}^t}$$
$$-\frac{2\mu}{\sigma^2}S_t\left(1+\frac{2\mu}{\sigma^2}\right)^{-1}\int_{-\infty}^{m_{t_0}^t}\exp\left(y+\frac{2\mu y}{\sigma^2}\right)\phi\left[y;-\mu\tau,\sigma\sqrt{\tau}\right]dy$$
$$+S_t\int_{-\infty}^{m_{t_0}^t}\frac{e^y}{\sqrt{2\pi\sigma^2\tau}}\exp\left[-\frac{(y-\mu\tau)^2}{2\sigma^2\tau}\right]dy$$

$$= S_t\int_{-\infty}^{m_{t_0}^t} e^y\phi\left[y;\mu\tau,\sigma\sqrt{\tau}\right]dy$$
$$+\frac{2\mu}{\sigma^2}S_t\left(1+\frac{2\mu}{\sigma^2}\right)^{-1}\left[\exp\left(m_{t_0}^t+\frac{2\mu m_{t_0}^t}{\sigma^2}\right)\Phi\left[\frac{m_{t_0}^t+\mu\tau}{\sigma\sqrt{\tau}}\right]\right]$$
$$-\frac{2\mu}{\sigma^2}S_t\left(1+\frac{2\mu}{\sigma^2}\right)^{-1}\int_{-\infty}^{m_{t_0}^t}e^y\phi\left[y;\mu\tau,\sigma\sqrt{\tau}\right]dy$$
$$+S_t\int_{-\infty}^{m_{t_0}^t} e^y\phi\left[y;\mu\tau,\sigma\sqrt{\tau}\right]dy.$$

Portanto,

$$E_Q\left[\inf_{t\le u\le T}(S_u)\times 1_{\left\{\inf_{t\le u\le T}(S_u)\le \inf_{t_0\le u\le t}(S_u)\right\}}\middle| F_t\right]$$

$$=\frac{2\mu}{\sigma^2}S_t\,\frac{\sigma^2}{\sigma^2+2\mu}\exp\left(m_{t_0}^t+\frac{2\mu m_{t_0}^t}{\sigma^2}\right)\Phi\left[\frac{m_{t_0}^t+\mu\tau}{\sigma\sqrt{\tau}}\right]$$
$$+S_t\left[2-\frac{2\mu}{\sigma^2+2\mu}\right]\int_{-\infty}^{m_{t_0}^t}e^y\phi\left[y;\mu\tau,\sigma\sqrt{\tau}\right]dy.$$

Por seu turno,

$$\int_{-\infty}^{m_{t_0}^t}e^y\phi\left[y;\mu\tau,\sigma\sqrt{\tau}\right]dy$$

$$=\int_{-\infty}^{m_{t_0}^t}\frac{e^y}{\sqrt{2\pi\sigma^2\tau}}\exp\left[-\frac{(y-\mu\tau)^2}{2\sigma^2\tau}\right]dy$$

$$=\int_{-\infty}^{m_{t_0}^t}\frac{1}{\sqrt{2\pi\sigma^2\tau}}\exp\left[-\frac{y^2-2y\mu\tau+(\mu\tau)^2-2\sigma^2\tau y}{2\sigma^2\tau}\right]dy$$

$$=\int_{-\infty}^{m_{t_0}^t}\frac{1}{\sqrt{2\pi\sigma^2\tau}}\exp\left[-\frac{y^2-2y\left(\mu\tau+\sigma^2\tau\right)+(\mu\tau)^2}{2\sigma^2\tau}\right]dy$$

$$=\int_{-\infty}^{m_{t_0}^t}\frac{\exp\left(-\frac{(\mu\tau)^2-\left(\mu\tau+\sigma^2\tau\right)^2}{2\sigma^2\tau}\right)}{\sqrt{2\pi\sigma^2\tau}}\exp\left[-\frac{y^2-2y\left(\mu\tau+\sigma^2\tau\right)+\left(\mu\tau+\sigma^2\tau\right)^2}{2\sigma^2\tau}\right]dy$$

$$=\exp\left(-\frac{(\mu\tau)^2-(\mu\tau)^2-2\mu\tau\sigma^2\tau-\left(\sigma^2\tau\right)^2}{2\sigma^2\tau}\right)\int_{-\infty}^{m_{t_0}^t}\phi\left[y;\mu\tau+\sigma^2\tau,\sigma\sqrt{\tau}\right]dy$$

$$=\exp\left(\mu\tau+\frac{\sigma^2\tau}{2}\right)\Phi\left[\frac{m_{t_0}^t-\mu\tau-\sigma^2\tau}{\sigma\sqrt{\tau}}\right].$$

Em síntese,

$$
\begin{aligned}
&L(-1)_t\left(S_t;\inf_{t_0\le u\le T}(S_u);T\right)\\
&= S_t e^{-q\tau}\\
&- e^{-r\tau} m_{t_0}^t \times\left\{\Phi\left[\frac{-m_{t_0}^t+\mu\tau}{\sigma\sqrt{\tau}}\right]-\exp\left(\frac{2\mu m_{t_0}^t}{\sigma^2}\right)\Phi\left[\frac{m_{t_0}^t+\mu\tau}{\sigma\sqrt{\tau}}\right]\right\}\\
&- e^{-r\tau} S_t \frac{2\mu}{\sigma^2+2\mu}\exp\left(m_{t_0}^t+\frac{2\mu m_{t_0}^t}{\sigma^2}\right)\Phi\left[\frac{m_{t_0}^t+\mu\tau}{\sigma\sqrt{\tau}}\right]\\
&- e^{-r\tau} S_t \frac{2\sigma^2+2\mu}{\sigma^2+2\mu}\exp\left(\mu\tau+\frac{\sigma^2\tau}{2}\right)\Phi\left[\frac{m_{t_0}^t-\mu\tau-\sigma^2\tau}{\sigma\sqrt{\tau}}\right].
\end{aligned}
$$

c) Defina uma carteira de activos (envolvendo opções) capaz de gerar daqui a 1 ano a média das cotações de fecho mensais registadas pelo índice SLB durante o próximo ano.

Carteira de opções a constituir (assumindo monitorização mensal do spot):

	Payoff daqui a 1 ano se $\bar{S} \le X$	Payoff daqui a 1 ano se $\bar{S} > X$
Long average price call	0	$\bar{S} - X$
Short average price put	$\bar{S} - X$	0
Aplicação fin. a 1ano e pelo present value do strike	+X	+X
Total:	$\bar{S}$	$\bar{S}$

CASO 2

a)

Taxa spot a 6M com capitalização continua:

$$r: \quad e^{r\times 6/12} = 1 + 4.75\% \times 6/12 \Rightarrow r = \frac{12}{6}\ln\left(1+4.75\%\times 6/12\right) \cong 4.694\%.$$

$$B_0 = 100\% \times e^{-4.694\%\times 0.5} + RV_0.$$

$$RV_{6M} = \begin{cases} 10\% \Leftarrow S_{6M} < 4{,}000 \times 0.95 = 3{,}800 \\ 0\% \Leftarrow ELSE \end{cases}$$

$$= 10\% \times D(-1)_{6M} \left(M = 1; S; X = 3{,}800; T = 6M\right).$$

Portanto,

$$RV_0 = 10\% \times D(-1)_0 \left(M = 1; S; X = 3{,}800; T = 6M\right)$$
$$= 10\% \times e^{-4.694\% \times 0.5} \times \Phi\left[-d_2^M(3{,}800)\right].$$

$$\Phi\left[-d_2^M(3{,}800)\right] = \Phi\left[-\frac{\ln\left(4{,}000 \big/ 3{,}800\right) + \left(4.694\% - 3\% - (0.15)^2 \big/ 2\right) \times 0.5}{0.15 \times \sqrt{0.5}}\right]$$
$$\cong \Phi(-0.5104) = 0.3049.$$

$$RV_0 = 10\% \times e^{-4.694\% \times 0.5} \times 0.3049 = 2.9780\%.$$

$$B_0 = 97.68\% + 2.978\% \cong 100.66\% > 100\% \Rightarrow \text{ Investir.}$$

b)

$$r: \quad e^{r \times 1} = 1 + 5\% \Rightarrow r = \ln(1 + 5\%) \cong 4.879\%.$$

Via proposição 20:

$$cp_0 = -4{,}000 \times e^{-3\% \times 1} \times M\left(-a_1^{**}, -b_1; \sqrt{\frac{0.5}{1}}\right) + 4{,}000 \times e^{-4.879\% \times 1} \times M\left(-a_2^{**}, -b_2; \sqrt{\frac{0.5}{1}}\right)$$
$$+ 500 \times e^{-4.879\% \times 0.5} \times \Phi\left(-a_2^{**}\right).$$

Visto que:

$$S^{**} = 3{,}475.49;$$

$$a_1^{**} = \frac{\ln\left(\frac{4{,}000}{3{,}475.49}\right) + \left(4.879\% - 3\% + \frac{(0.15)^2}{2}\right)\times 0.5}{0.15\times\sqrt{0.5}} \cong 1.46679932;$$

$$a_2^{**} = 1.46679932 - 0.15\times\sqrt{0.5} \cong 1.360733;$$

$$b_1 = \frac{\ln\left(\frac{4{,}000}{4{,}000}\right) + \left(4.879\% - 3\% + \frac{(0.15)^2}{2}\right)\times 1}{0.15\times\sqrt{1}} \cong 0.20026776;$$

$$b_2 = 0.20026776 - 0.15\times\sqrt{1} \cong 0.050268;$$

então:

$$\begin{aligned} cp_0 &= -4{,}000\times e^{-3\%\times 1}\times M(-1.46679932,-0.20026776;0.7071068) \\ &+ 4{,}000\times e^{-4.879\%\times 1}\times M(-1.360733,-0.050268;0.7071068) \\ &+ 500\times e^{-4.879\%\times 0.5}\times \Phi(-1.360733). \end{aligned}$$

Utilizando a tabela de probabilidades inclusa no enunciado e a distribuição normal e univariada standard,

$$\begin{aligned} cp_0 &= -4{,}000\times e^{-3\%\times 1}\times 0.06675 \\ &+ 4{,}000\times e^{-4.879\%\times 1}\times 0.082458 \\ &+ 500\times e^{-4.879\%\times 0.5}\times 0.086799 \\ &= 12.66. \end{aligned}$$

c)

$$B_0 = 100\%\times e^{-4.879\%\times 1} + RV_0.$$

Assumindo taxas de juro constantes para os próximos 12 meses,

$$RV_0 = E_Q\left(10\%\times e^{-4.879\%\times v}\,\middle|\,F_0\right),$$

sendo "v" o primeiro tempo de passagem da cotação do índice subjacente pela barreira 4,000 x 0.9 = 3,600.

Trata-se portanto do present value de um knock-out non-deferrable rebate igual a 10% e associado a uma down barrier igual a 3,600 pontos de índice. Utilizando a equação (131) com $\eta = -1$:

$$\mu = 4.879\% - 3\% - \frac{(0.15)^2}{2} \cong 0.00754.$$

$$\psi = \sqrt{(0.00754)^2 + 2 \times (0.15) \times 4.879\%^2} \cong 0.04746.$$

$$RV_0 = 10\% \times \left\{ \left(\frac{3,600}{4,000}\right)^{\frac{0.00754\text{-}0.04746}{(0.15)^2}} \times \Phi\left[\frac{\ln\left(\frac{3,600}{4,000}\right) - 0.04746 \times 1}{0.15 \times \sqrt{1}}\right] \right.$$

$$\left. + \left(\frac{3,600}{4,000}\right)^{\frac{0.00754+0.04746}{(0.15)^2}} \times \Phi\left[\frac{\ln\left(\frac{3,600}{4,000}\right) + 0.04746 \times 1}{0.15 \times \sqrt{1}}\right] \right\}$$

$$RV_0 = 10\% \times \left\{ \left(\frac{3,600}{4,000}\right)^{\frac{0.00754\text{-}0.04746}{(0.15)^2}} \times \Phi[\text{-}1.0188] + \left(\frac{3,600}{4,000}\right)^{\frac{0.00754+0.04746}{(0.15)^2}} \times \Phi[\text{-}0.38601] \right\}$$

$$= 10\% \times \left\{ \left(\frac{3,600}{4,000}\right)^{\frac{0.00754\text{-}0.04746}{(0.15)^2}} \times 0.154149 + \left(\frac{3,600}{4,000}\right)^{\frac{0.00754+0.04746}{(0.15)^2}} \times 0.349746 \right\}$$

$$\cong 10\% \times 0.456168$$

$$\cong 4.56\%.$$

Portanto,

$B_0 = 95.24\% + 4.56\% \cong 99.80\% < 100\% \Rightarrow$ Não investir.

d)

$B_0 = 100\% \times e^{-4.879\% \times 1} + RV_0.$

Valor terminal da remuneração variável:

$$RV_{12M} = 20\% \times \begin{cases} \dfrac{5{,}000 - \inf\limits_{t \le u \le T}(S_u)}{5{,}000} \Leftarrow \inf\limits_{-24M \le u \le 12M}(S_u) < 5{,}000 \\ 0\% \Leftarrow \inf\limits_{-24M \le u \le 12M}(S_u) \ge 5{,}000 \end{cases}$$

$$\Updownarrow$$

$$RV_{12M} = \frac{20\%}{5{,}000} \times \left[5{,}000 - \inf_{-24M \le u \le 12M}(S_u)\right]^+$$

$$\Updownarrow$$

$$RV_{12M} = \frac{20\%}{5{,}000} \times L(-1)_{12M}\left[\inf_{-24M \le u \le 12M}(S_u); X = 5{,}000; T = 12M\right]$$

Portanto, o valor actual da remuneração variável pode ser escrito em função do valor actual de uma fixed strike lookback put com strike igual a 5,000.00 pontos e vencimento daqui a 12 meses:

$$RV_0 = \frac{20\%}{5{,}000} \times L(-1)_0\left[\inf_{-24M \le u \le 12M}(S_u); X = 5{,}000; T = 12M\right]$$

Utilizando a penúltima equação da página 61 dos apontamentos, então

$$RV_0 = \frac{20\%}{5{,}000} \times \left\{e^{-4.879\% \times 1} \times (5{,}000 - 3{,}900) + L(-1)_0\left[\inf_{0 \le u \le 12M}(S_u); X = 3{,}900; T = 12M\right]\right\}$$

Com base nas cotações disponibilizadas no enunciado,

$$RV_0 = \frac{20\%}{5{,}000} \times \left\{e^{-4.879\% \times 1} \times (5{,}000 - 3{,}900) + 315.63\right\} \cong 5.45\%.$$

Portanto,

$$B_0 = 95.24\% + 5.45\% \cong 100.69\% > 100\% \Rightarrow \text{ Investir.}$$

<u>CASO 3</u>

a)

$$r: \quad e^{r \times 1} = 1 + 5\% \Rightarrow r = \ln(1 + 5\%) \cong 4.879\%.$$

$$S_{2,8} = 3\,648,71 \times \exp\left\{\left[4.879\% - 3\% - \frac{(0.15)^2}{2}\right] \times \frac{2}{12} + (-0.3629) \times 0.15 \times \sqrt{\frac{2}{12}}\right\}$$

$\cong 3\,573,01.$

$$\min_{i=1,\ldots,6}(S_{i,6}) = 4,022.13 > 3{,}800 \Rightarrow \quad V_{6,6} = 0.00\,.$$

$$(V_{6,6})^2 = (0.00\;)^2 \cong 0.00\,.$$

$$\min_{i=1,\ldots,6}(S_{i,7}) = 3,617.65 < 3{,}800 \Rightarrow \quad V_{6,7} = \max(\,4{,}000 - 3\,{,}617.65\,;0) = 382.35\;\;.$$

$$(V_{6,7})^2 = (382{,}35)^2 \cong 146\,{,}191.41.$$

b)

$$\sum_{j=1}^{10} V_{T,j} = 994.32 + 1{,}324.11 + 382.25 + 1{,}129.62 = 3{,}830.40.$$

$$\hat{V}_0 = e^{-4.879\% \times 1} \times \frac{3{,}830.40}{10} \cong 364.80\,.$$

$$\sum_{j=1}^{10} (V_{T,j})^2 = 988{,}677.14 + 1{,}753{,}261.63 + 146{,}191.41 + 1{,}276{,}040.11$$

$= 4{,}164{,}170.28$

$$\sigma(\hat{V}_0) = \frac{e^{-4.879\% \times 1}}{\sqrt{10}} \times \sqrt{\frac{4{,}164{,}170.28 - {(3{,}830.40)^2}/{10}}{10 - 1}} \cong 164{,}86.$$

c)

Via equação (106),

$$DI(-1)_0\left(S; X = S_0 = 4{,}000; L = 3{,}800; R = 0; T = 12M\right)$$

$$= \left(\frac{3{,}800}{4{,}000}\right)^{\frac{2\mu}{(0.15)^2}} \times \left[c_0\left(S = \frac{(3{,}800)^2}{4{,}000}; X = 4{,}000; T = 12M\right) - c_0\left(S = \frac{(3{,}800)^2}{4{,}000}; X = 3{,}800; T = 12M\right)\right.$$

$$\left. + (4{,}000 - 3{,}800) \times e^{-4.879\% \times 1} \times \Phi\left(d_2^M(3{,}800; 4{,}000)\right)\right]$$

$$+ p_0\left(S = 4{,}000; X = 3{,}800; T = 12M\right) + (4{,}000 - 3{,}800) \times e^{-4.879\% \times 1} \times \Phi\left(-d_2^M(4{,}000; 3{,}800)\right).$$

Visto que:

$$\mu = 4.879\% - 3\% - \frac{(0.15)^2}{2} \cong 0.00754;$$

$$p_0\left(S = 4{,}000; X = 3{,}800; T = 12M\right) = 116.83;$$

e que uma opção financeira é uma função homogénea de grau 1 no *spot* e no *strike*,

$$c_0\left(S = \frac{(3{,}800)^2}{4{,}000}; X = 4{,}000; T = 12M\right)$$

$$= \frac{(3{,}800)^2}{(4{,}000)^2} \times c_0\left(S = 4{,}000; X = \frac{(4{,}000)^3}{(3{,}800)^2} \cong 4{,}432.13; T = 12M\right)$$

$$= \frac{(3{,}800)^2}{(4{,}000)^2} \times 109.31 \cong 98.66;$$

$$c_0\left(S = \frac{(3{,}800)^2}{4{,}000}; X = 3{,}800; T = 12M\right)$$

$$= \frac{(3{,}800)^2}{(4{,}000)^2} \times c_0\left(S = 4{,}000; X = \frac{(4{,}000)^2}{3{,}800} \cong 4{,}210.53; T = 12M\right)$$

$$= \frac{(3{,}800)^2}{(4{,}000)^2} \times 177.29 \cong 160.01;$$

$$\Phi\left[d_2^M(3{,}800;4{,}000)\right]=\Phi\left[\frac{\ln\left(3{,}800/4{,}000\right)+\left(4.879\%-3\%-(0.15)^2/2\right)\times 1}{0.15\times\sqrt{1}}\right]$$

$$\cong\Phi(-0.2917)=0.3853;$$

$$\Phi\left[-d_2^M(4{,}000;3{,}800)\right]=\Phi\left[-\frac{\ln\left(4{,}000/3{,}800\right)+\left(4.879\%-3\%-(0.15)^2/2\right)\times 1}{0.15\times\sqrt{1}}\right]$$

$$\cong\Phi(-0.3922)=0.3474;$$

Então:

$$DI(-1)_0(S;X=S_0=4{,}000;L=3{,}800;R=0;T=12M)$$

$$=\left(\frac{3{,}800}{4{,}000}\right)^{\frac{2\times 0.00754}{(0.15)^2}}\times\left[98.66-160.01+(4{,}000-3{,}800)\times e^{-4.879\%\times 1}\times 0.3853\right]$$

$$+116.83+(4{,}000-3{,}800)\times e^{-4.879\%\times 1}\times 0.3474$$

$$\cong 194.64.$$