OPÇÕES EXÓTICAS
MSc MATEMÁTICA FINANCEIRA 2008/09
EXAME - Resolução

29/07/09 **Duração: 2.5 horas**

<u>CASO 1</u>

a) Considere uma put Europeia sobre o activo "S", com strike "X", com vencimento no momento "T" e com um prémio igual a "Kp". Ao contrário do que é habitual, o prémio "kp" não é liquidado hoje na data de transacção (momento "t"). Este prémio apenas será liquidado pelo comprador da put na data de vencimento (momento "T") caso a put termine in-the-money. Formule, no momento "t" ($\leq T$), o cálculo do prémio "kp" desta put option.

Trata-se de uma paylater put. O seu payoff terminal é dado por:

$$p_T^{PL} = \begin{cases} X - S_T - kp \Leftarrow S_T < X \\ 0 \Leftarrow else \end{cases}$$

$$= \begin{cases} X - S_T \Leftarrow S_T < X \\ 0 \Leftarrow else \end{cases} - \begin{cases} kp \Leftarrow S_T < X \\ 0 \Leftarrow else \end{cases}$$

$$= p_T(S_T, X, T) - D(-1)_T(S_T, X, T; M = kp).$$

Portanto,

$$p_t^{PL} = p_t(S_t, X, T) - kp \times D(-1)_t(S_t, X, T; M = 1).$$

Como não há qualquer liquidação do prémio hoje, então $p_t^{PL} = 0$ e portanto,

$$0 = p_t(S_t, X, T) - kp \times D(-1)_t(S_t, X, T; M = 1)$$

$$\Rightarrow kp = \frac{p_t(S_t, X, T)}{D(-1)_t(S_t, X, T; M = 1)}.$$

b) Considere uma fixed-strike lookback European call com strike "X", vencimento no momento "T" e que inside sobre apenas 80% do máximo registado pelo activo subjacente "S" entre os momentos "t" (data de avaliação) e "T". Defina, no momento "t" ($\leq T$), a fórmula de avaliação deste contrato.

O *payoff*, na data de vencimento, da lookback call em apreço é dado por:

$$V_T = \left[80\% \times \sup_{u \in [t,T]} (S_u) - X \right]^+ .$$

Trata-se daquilo que se designa por uma "partial" lookback fixed-strike call.

Portanto,

$$V_t = e^{-r(T-t)} E_Q \left\{ \left[80\% \times \sup_{u \in [t,T]} (S_u) - X \right]^+ \middle| F_t \right\}$$

$$= 80\% \times e^{-r(T-t)} E_Q \left\{ \left[\sup_{u \in [t,T]} (S_u) - \frac{X}{0.8} \right]^+ \middle| F_t \right\}$$

$$= 80\% \times L(1)_t \left(\sup_{u \in [t,T]} (S_u); \frac{X}{0.8}; T \right).$$

A "full" fixed-strike lookback call pode, por seu turno, ser avaliada via equação (155) dos apontamentos.

c) Considere uma call Europeia sobre o activo "S", com strike "X", com vencimento no momento "T" e com um payoff terminal igual a:

$$V_T = \max[\min(S_T, L) - X; 0],$$

onde $L \in \Re_+$ estabelece uma limitação sobre o potencial de valorização da call. Defina, no momento "t" ($\leq T$), a fórmula de avaliação deste contrato.

Trata-se de uma capped call.

Só faz sentido considerar que L>X, pois caso contrário o payoff seria sempre negativo.

Então,

$$V_T = \max[\min(S_T, L) - X; 0]$$

$$= \begin{cases} \max[S_T - X; 0] \Leftarrow S_T < L \\ \max[L - X; 0] \Leftarrow S_T \geq L \end{cases}$$

$$= \begin{cases} S_T - X \Leftarrow S_T < L \wedge S_T > X \\ L - X \Leftarrow S_T \geq L \\ 0 \Leftarrow S_T < X \end{cases}$$

$$= \begin{cases} S_T - X \Leftarrow X < S_T < L \\ L - X \Leftarrow S_T \geq L \\ 0 \Leftarrow S_T < X \end{cases}$$

$$= \begin{cases} S_T - X \Leftarrow S_T > X \\ 0 \Leftarrow S_T \leq X \end{cases} - \begin{cases} S_T - L \Leftarrow S_T > L \\ 0 \Leftarrow S_T \leq L \end{cases}$$

$$= c_T(S_T; X; T) - c_T(S_T; L; T).$$

Portanto,

$$V_t = c_t(S_t; X; T) - c_t(S_t; L; T).$$

CASO 2

a)

$$r: \quad e^{r \times 6/12} = 1 + 1.5\% \times 6/12 \Rightarrow r = \frac{12}{6}\ln\left(1 + 1.5\% \times 6/12\right) \cong 1.494\%.$$

$$S_{4,7} = 1{,}081.17 \times \exp\left\{\left[1.494\% - 3.475\% - \frac{(0.3)^2}{2}\right] \times \frac{1}{12} + (-0.7253) \times 0.3 \times \sqrt{\frac{1}{12}}\right\}$$

$$\Rightarrow S_{4,7} \cong 1{,}009.88.$$

$$V_{T,6} = \max(900 - 823.64; 0) = 76.36.$$

$\left(V_{T,6}\right)^2 = (76.36)^2 \cong 5{,}831.41.$

$V_{T,7} = \max(900 - 908.48; 0) = 0.$

$\left(V_{T,7}\right)^2 = (0)^2 = 0.$

b)

$$\hat{V}_0 = e^{-1.494\% \times 6/12} \times \frac{577.59}{10} \cong 57.33.$$

$$\sigma\left(V_{t,j}\right) = e^{-1.494\% \times 6/12} \sqrt{\frac{84{,}756.52 - \frac{(577.59)^2}{10}}{9}} \cong 75.01.$$

$$\sigma\left(\hat{V}_0\right) = \frac{75.01}{\sqrt{10}} \cong 23.72.$$

c)

$V_{t,3} = \max(1{,}026.93 - 950; 0) = 76.36.$

$$\hat{V}_0 = \frac{76.36 + 219.45 + 8.27 + 42.70}{10} \cong 34.68.$$

CASO 3

a)

$B_0 = 100\% \times e^{-2\% \times 1} + RV_0.$

$$RV_1 = \begin{cases} 15\% \Leftarrow 7{,}500 \le S_1 \le 8{,}000 \\ 0\% \Leftarrow ELSE \end{cases}$$

$$= 15\% \times RD_1(S; X_a = 7{,}500; X_b = 8{,}000; T = 1; M = 1).$$

Portanto,

$$RV_0 = 15\% \times RD_0(S; X_a = 7{,}500; X_b = 8{,}000; T = 1; M = 1).$$

Mas,

$$RD_0(S; X_a = 7{,}500; X_b = 8{,}000; T = 1; M = 1)$$

$$= e^{-2\% \times 1} \times \{\Phi[d_2^M(7{,}500)] - \Phi[d_2^M(8{,}000)]\}.$$

$$d_2^M(7{,}500) = \frac{\ln\left(7{,}205/7{,}500\right) + \left(2\% - 1\% - (0.2)^2/2\right) \times 1}{0.2 \times \sqrt{1}} \cong \text{-}0.2506$$

$$\Rightarrow \Phi[d_2^M(7{,}500)] = \Phi(-0.2506) = 0.4010.$$

$$d_2^M(8{,}000) = \frac{\ln\left(7{,}205/8{,}000\right) + \left(2\% - 1\% - (0.2)^2/2\right) \times 1}{0.2 \times \sqrt{1}} \cong \text{-}0.5733$$

$$\Rightarrow \Phi[d_2^M(8{,}000)] = \Phi(\text{-}0.5733) = 0.2832.$$

Em suma,

$$RV_0 = 15\% \times e^{-2\% \times 1} \times (0.4010 - 0.2832) \cong 1.73\%.$$

$B_0 = 98.02\% + 1.73\% = 99.75\% < 100\% \Rightarrow$ Não depositar.

b)

$$B_0 = 80\% \times e^{-3\% \times 1} + RV_0.$$

$$RV_1 = \begin{cases} \frac{1}{0.9S_0} c_1(S_1; X = 0.9S_0; 2y) \Leftarrow S_1 > 6{,}760.18 \\ \frac{1}{S_0} p_1(S_1; X = S_0; 2y) \Leftarrow S_1 < 6{,}760.18 \end{cases}$$

$$= \frac{1}{0.9S_0} \begin{cases} c_1(S_1; X = 0.9S_0; 2y) - 0 \Leftarrow S_1 > 6{,}760.18 \\ 0 \Leftarrow S_1 < 6{,}760.18 \end{cases} + \frac{1}{S_0} \begin{cases} 0 \Leftarrow S_1 > 6{,}760.18 \\ p_1(S_1; X = S_0; 2y) - 0 \Leftarrow S_1 < 6{,}760.18 \end{cases}$$

$$= \frac{1}{0.9S_0} c_1[c_1(S_1; X = 0.9S_0; 2y); 0; 1y] + \frac{1}{S_0} c_1[p_1(S_1; X = S_0; 2y); 0; 1y].$$

Portanto,

$$RV_0 = \frac{1}{0.9S_0} c_0[c_0(S_0; X = 0.9S_0; 2y); 0; 1y] + \frac{1}{S_0} c_0[p_0(S_0; X = S_0; 2y); 0; 1y].$$

Via proposição 19:

$$c_0[c_0(S_0; X = 0.9S_0; 2y); 0; 1y]$$

$$= 7{,}205 \times e^{-1\% \times 2} \times M\left(a_1^*, b_1; \sqrt{\frac{1}{2}}\right) - 0.9 \times 7{,}205 \times e^{-3\% \times 2} \times M\left(a_2^*, b_2; \sqrt{0.5}\right).$$

Visto que:

$S^* = 6{,}760.18$;

$$a_1^* = \frac{\ln\left(\frac{7{,}205}{6{,}760.18}\right) + \left(3\% - 1\% + \frac{(0.2)^2}{2}\right) \times 1}{0.2 \times \sqrt{1}} \cong 0.518627;$$

$a_2^* = 0.518627 - 0.2 \times \sqrt{1} \cong 0.31862676$;

$$b_1 = \frac{\ln\left(\frac{7{,}205}{0.9\times 7{,}205}\right)+\left(3\% - 1\% + \frac{(0.2)^2}{2}\right)\times 2}{0.2\times\sqrt{2}} \cong 0.655348;$$

$$b_2 = 0.655348 - 0.2\times\sqrt{2} \cong 0.372505675;$$

então:

$$c_0[c_0(S_0; X = 0.9S_0; 2y); 0; 1y] = 7{,}205\times e^{-1\%\times 2}\times M(0.518627, 0.655348; 0.707106781)$$
$$- 0.9\times 7{,}205\times e^{-3\%\times 2}\times M(0.31862676, 0.372505675; 0.707106781).$$

Utilizando a tabela de probabilidades,

$$c_0[c_0(S_0; X = 0.9S_0; 2y); 0; 1y] = 7{,}205\times e^{-1\%\times 2}\times 0.615148$$
$$- 0.9\times 7{,}205\times e^{-3\%\times 2}\times 0.5174816$$

$$\cong 1{,}184.18.$$

Via proposição 20:

$$c_0[p_0(S_0; X = S_0; 2y); 0; 1y]$$

$$= -7{,}205\times e^{-1\%\times 2}\times M\left(-a_1^{**}, -b_1; \sqrt{\frac{1}{2}}\right) + 7{,}205\times e^{-3\%\times 2}\times M\left(-a_2^{**}, -b_2; \sqrt{0.5}\right).$$

Visto que:

$$S^{**} = 6{,}760.18;$$

$$a_1^{**} = \frac{\ln\left(\frac{7{,}205}{6{,}760.18}\right)+\left(3\% - 1\% + \frac{(0.2)^2}{2}\right)\times 1}{0.2\times\sqrt{1}} \cong 0.518627;$$

$$a_2^{**} = 0.518627 - 0.2\times\sqrt{1} \cong 0.31862676;$$

$$b_1 = \frac{\ln\left(\frac{7,205}{7,205}\right) + \left(3\% - 1\% + \frac{(0.2)^2}{2}\right) \times 2}{0.2 \times \sqrt{2}} \cong 0.28284271;$$

$$b_2 = 0.28284271 - 0.2 \times \sqrt{2} \cong 0;$$

então:

$$c_0[p_0(S_0; X = S_0; 2y); 0; 1y] = -7,205 \times e^{-1\% \times 2} \times M(-0.518627, -0.28284271; 0.707107)$$
$$+ 7,205 \times e^{-3\% \times 2} \times M(-0.31862676, 0; 0.707107).$$

Utilizando a tabela de probabilidades,

$$c_0[p_0(S_0; X = S_0; 2y); 0; 1y] = -7,205 \times e^{-1\% \times 2} \times 0.225529$$
$$+ 7,205 \times e^{-3\% \times 2} \times 0.304692$$

$$\cong 474.70.$$

Em sintese,

$$RV_0 = \frac{1}{0.9 \times 7,205} \times 1,184.18 + \frac{1}{7,205} \times 474.70 \cong 24.85\%.$$

e

$$B_0 = 75.34\% + 24.85\% \cong 100.19\% > 100\% \Rightarrow \text{ Depositar.}$$

c)

$$B_0 = 100\% \times e^{-2\% \times 1} + RV_0.$$

$$RV_1 = \begin{cases} 1\% \Leftarrow \sup\limits_{0<u\leq 1}(S_u) \geq 8,646 \\ 60\% \times \frac{S_0 - S_1}{S_0} \Leftarrow \frac{S_0 - S_1}{S_0} \geq 0\% \wedge \sup\limits_{0<u\leq 1}(S_u) \leq 8,646 \\ 0\% \Leftarrow \frac{S_0 - S_1}{S_0} \leq 0\% \wedge \sup\limits_{0<u\leq 1}(S_u) \leq 8,646 \end{cases}$$

$$\Leftrightarrow RV_1 = \frac{60\%}{S_0} \times \begin{cases} \frac{0.01S_0}{0.6} \Leftarrow \sup_{0<u\leq 1}(S_u) \geq 8{,}646 \\ S_0 - S_1 \Leftarrow S_1 \leq S_0 \wedge \sup_{0<u\leq 1}(S_u) \leq 8{,}646 \\ 0 \Leftarrow S_1 > S_0 \wedge \sup_{0<u\leq 1}(S_u) \leq 8{,}646 \end{cases}$$

$$= \frac{60\%}{S_0} \times UO(-1)_1(S_1; X = S_0; U = 8{,}646; R = 0.01(6)S_0; T = 1y).$$

Portanto,

$$RV_0 = \frac{60\%}{7{,}205} \times UO(-1)_0(S_0; X = 7{,}205; U = 8{,}646; R = 0.01(6)S_0; T = 1y).$$

Por outro lado,

$$UO(-1)_0(S_0; X = 7{,}205; U = 8{,}646; R = 0.01S_0; T = 1y)$$

$$= UO(-1)_0(S_0; X = 7{,}205; U = 8{,}646; R = 0; T = 1y) + KOR(1)_0,$$

onde $KOR(1)_0$ designa o valor actual de um deferrable knock-out rebate com up barrier.

Começando pela up-and-out put sem rebate e via Proposição 43,

$$UO(-1)_0(S_0; X = 7{,}205; U = 8{,}646; R = 0; T = 1y)$$

$$= p_0(S_0; 7{,}205; T = 1y) - \left(\frac{8{,}646}{7{,}205}\right)^{\frac{2\mu}{\sigma^2}} p_0\left(\frac{(8{,}646)^2}{7{,}205}; 7{,}205; T = 1y\right)$$

$$= p_0(S_0; 7{,}205; T = 1y) - \left(\frac{8{,}646}{7{,}205}\right)^{\frac{2\mu}{\sigma^2}} \frac{(8{,}646)^2}{(7{,}205)^2} p_0\left(7{,}205; \frac{(7{,}205)^3}{(8{,}646)^2} = 5{,}003.47; T = 1y\right).$$

Visto que

$$\mu = 2\% - 1\% - \frac{(0.2)^2}{2} \cong \text{-0.01},$$

e utilizando os dados do enunciado, então

$$UO(-1)_0\left(S_0; X = 7{,}205; U = 8{,}646; R = 0; T = 1y\right)$$

$$= 530.60 - \left(\frac{8{,}646}{7{,}205}\right)^{\frac{2\times(-0.01)}{0.2^2}} \frac{(8{,}646)^2}{(7{,}205)^2} \times 13.96 \cong 512.24.$$

Relativamente ao rebate,

$$KOR(1)_0 = 0.01(6)S_0 \times e^{-2\%\times 1} \times Q\left[\sup_{0<u\leq 1}(S_u) \geq 8{,}646\right]$$

$$= 0.01(6)S_0 \times e^{-2\%\times 1} \times Q\left[\sup_{0<u\leq 1}\left(\ln\left(\frac{S_u}{S_0}\right)\right) \geq \ln\left(\frac{8{,}646}{S_0}\right)\right]$$

$$= 0.01(6)S_0 \times e^{-2\%\times 1} \times \left\{1 - Q\left[\sup_{0<u\leq 1}\left(\ln\left(\frac{S_u}{S_0}\right)\right) < \ln\left(\frac{8{,}646}{S_0}\right)\right]\right\}.$$

Via Proposição 45,

$$Q\left[\sup_{0<u\leq 1}\left(\ln\left(\frac{S_u}{S_0}\right)\right) < \ln\left(\frac{8{,}646}{S_0}\right)\right]$$

$$= \Phi\left[-d_2^M(7{,}205; 8{,}646)\right] - \left(\frac{8{,}646}{7{,}205}\right)^{\frac{2\times(-0.01)}{0.2^2}} \Phi\left[-d_2^M(8{,}646; 7{,}205)\right]$$

$$= \Phi\left[-\frac{\ln\left(\frac{7{,}205}{8{,}646}\right) + \left(2\% - 1\% - \frac{(0.2)^2}{2}\right)\times 1}{0.2\times\sqrt{1}}\right]$$

$$- \left(\frac{8{,}646}{7{,}205}\right)^{\frac{2\times(-0.01)}{0.2^2}} \Phi\left[-\frac{\ln\left(\frac{8{,}646}{7{,}205}\right) + \left(2\% - 1\% - \frac{(0.2)^2}{2}\right)\times 1}{0.2\times\sqrt{1}}\right]$$

$$= \Phi(0.9616) - \left(\frac{8{,}646}{7{,}205}\right)^{\frac{2\times(-0.01)}{0.2^2}} \Phi(-0.8616)$$

$$\cong 0.6544.$$

Portanto,

$$KOR(1)_0 = 0.01(6)S_0 \times e^{-2\%\times 1} \times Q\left[\sup_{0<u\leq 1}(S_u) \geq 8{,}646\right]$$

$$= 0.01(6) \times 7{,}205 \times e^{-2\%\times 1} \times \{1 - 0.6544\}$$

$$\cong 40.68.$$

Em suma,

$$UO(-1)_0(S_0; X = 7{,}205; U = 8{,}646; R = 0.01S_0; T = 1y)$$

$$= 512.24 + 40.68$$

$$\cong 552.93.$$

$$RV_0 = \frac{60\%}{7{,}205} \times 552.93 \cong 4.60\%.$$

$B_0 = 98.02\% + 4.60\% = 102.62\% > 100\% \Rightarrow$ Depositar.

d)

$B_0 = 100\% \times e^{-3\%\times 2} + RV_0.$

Se daqui a 1 ano o obrigacionista optar pela valorização, a RV daqui a 2 anos será igual a:

$$RV_{24M}^{v} = x\% \times \begin{cases} \dfrac{S_{24M} - S_0}{S_0} \Leftarrow S_{24M} > 1.2S_0 \\ 0\% \Leftarrow S_{24M} \leq 1.2S_0 \end{cases}$$

$$= \frac{x\%}{S_0} \times \begin{cases} S_{24M} - S_0 \Leftarrow S_{24M} > 1.2S_0 \\ 0\% \Leftarrow S_{24M} \leq 1.2S_0 \end{cases}$$

$$= \frac{x\%}{S_0} \times \left(\begin{cases} S_{24M} - 1.2S_0 \Leftarrow S_{24M} > 1.2S_0 \\ 0 \Leftarrow S_{24M} \leq 1.2S_0 \end{cases} + \begin{cases} 0.2S_0 \Leftarrow S_{24M} > 1.2S_0 \\ 0 \Leftarrow S_{24M} \leq 1.2S_0 \end{cases} \right)$$

$$= \frac{x\%}{S_0} \times \left(c_{24M}\left(S_{24M}; 1.2S_0; 24M\right) + c_{24M}^{d}\left(S_{24M}; 1.2S_0; 24M; M = 0.2S_0\right)\right).$$

O valor desta RV daqui a 1 ano é dado por:

$$RV_{12M}^{v} = \frac{x\%}{S_0} \times \left(c_{12M}\left(S_{12M}; 1.2S_0; 24M\right) + c_{12M}^{d}\left(S_{12M}; 1.2S_0; 24M; M = 0.2S_0\right)\right).$$

De forma similar, se daqui a 1 ano o obrigacionista optar pela desvalorização, a RV daqui a 2 anos será igual a:

$$RV_{24M}^{d} = x\% \times \begin{cases} \dfrac{S_0 - S_{24M}}{S_0} \Leftarrow S_{24M} \leq 1.2S_0 \\ 0\% \Leftarrow S_{24M} > 1.2S_0 \end{cases}$$

$$= \frac{x\%}{S_0} \times \begin{cases} S_0 - S_{24M} \Leftarrow S_{24M} \leq 1.2S_0 \\ 0\% \Leftarrow S_{24M} > 1.2S_0 \end{cases}$$

$$= \frac{x\%}{S_0} \times \left(\begin{cases} 1.2S_0 - S_{24M} \Leftarrow S_{24M} \leq 1.2S_0 \\ 0 \Leftarrow S_{24M} > 1.2S_0 \end{cases} - \begin{cases} 0.2S_0 \Leftarrow S_{24M} \leq 1.2S_0 \\ 0 \Leftarrow S_{24M} > 1.2S_0 \end{cases} \right)$$

$$= \frac{x\%}{S_0} \times \left(p_{24M}\left(S_{24M}; 1.2S_0; 24M\right) - p_{24M}^{d}\left(S_{24M}; 1.2S_0; 24M; M = 0.2S_0\right)\right).$$

O valor desta RV daqui a 1 ano é dado por:

$$RV_{12M}^{d} = \frac{x\%}{S_0} \times \left(p_{12M}\left(S_{12M}; 1.2S_0; 24M\right) - p_{12M}^{d}\left(S_{12M}; 1.2S_0; 24M; M = 0.2S_0\right)\right).$$

Em suma, o valor da RV daqui a 1 ano corresponderá à melhor das duas alternativas:

$$RV_{12M} = \max\left(RV_{12M}^{v}; RV_{12M}^{d}\right)$$

$$= \frac{x\%}{S_0} \times \max\left[c_{12M}\left(S_{12M}; 1.2S_0; 24M\right) + c_{12M}^{d}\left(S_{12M}; 1.2S_0; 24M; M = 0.2S_0\right);\right.$$
$$\left. p_{12M}\left(S_{12M}; 1.2S_0; 24M\right) - p_{12M}^{d}\left(S_{12M}; 1.2S_0; 24M; M = 0.2S_0\right)\right]$$

Utilizando a paridade put-call,

$$p_{12M}\left(S_{12M}; 1.2S_0; 24M\right) = c_{12M}\left(S_{12M}; 1.2S_0; 24M\right) - S_{12M} \times e^{-1\% \times 1} + 1.2S_0 \times e^{-4\% \times 1},$$

bem como as fórmulas de avaliação de *cash-or-nothing* options,

$$c_{12M}^{d}\left(S_{12M}; 1.2S_0; 24M; M = 0.2S_0\right) = 0.2S_0 \times e^{-4\% \times 1} \times \Pr\left(S_{12M} > 1.2S_0\right),$$

$$p_{12M}^{d}\left(S_{12M}; 1.2S_0; 24M; M = 0.2S_0\right) = 0.2S_0 \times e^{-4\% \times 1} \times \Pr\left(S_{12M} < 1.2S_0\right),$$

então:

$$RV_{12M} = \max\left(RV_{12M}^{v}; RV_{12M}^{d}\right)$$

$$= \frac{x\%}{S_0} \times \max\left[c_{12M}\left(S_{12M};1.2S_0;24M\right) + 0.2S_0 \times e^{-4\%\times1} \times \Pr\left(S_{12M} > 1.2S_0\right);\right.$$
$$\left. c_{12M}\left(S_{12M};1.2S_0;24M\right) - S_{12M} \times e^{-1\%\times1} + 1.2S_0 \times e^{-4\%\times1} - 0.2S_0 \times e^{-4\%\times1} \times \Pr\left(S_{12M} < 1.2S_0\right)\right]$$

$$= \frac{x\%}{S_0} \times \left\{c_{12M}\left(S_{12M};1.2S_0;24M\right) + \max\left[0.2S_0 \times e^{-4\%\times1} \times \Pr\left(S_{12M} > 1.2S_0\right);\right.\right.$$
$$\left.\left. - S_{12M} \times e^{-1\%\times1} + 1.2S_0 \times e^{-4\%\times1} - 0.2S_0 \times e^{-4\%\times1} \times \left(1 - \Pr\left(S_{12M} > 1.2S_0\right)\right)\right]\right\}$$

$$= \frac{x\%}{S_0} \times \left\{c_{12M}\left(S_{12M};1.2S_0;24M\right) + 0.2S_0 \times e^{-4\%\times1} \times \Pr\left(S_{12M} > 1.2S_0\right)\right.$$
$$\left. + \max\left[0; -S_{12M} \times e^{-1\%\times1} + 1.2S_0 \times e^{-4\%\times1} - 0.2S_0 \times e^{-4\%\times1}\right]\right\}$$

$$= \frac{x\%}{S_0} \times \left\{c_{12M}\left(S_{12M};1.2S_0;24M\right) + c_{12M}^{d}\left(S_{12M};1.2S_0;24M; M = 0.2S_0\right)\right.$$
$$\left. + e^{-1\%\times1} \times \max\left[0; -S_{12M} + S_0 \times e^{-4\%\times1} \times e^{1\%\times1}\right]\right\}$$

$$= \frac{x\%}{S_0} \times \left\{c_{12M}\left(S_{12M};1.2S_0;24M\right) + c_{12M}^{d}\left(S_{12M};1.2S_0;24M; M = 0.2S_0\right)\right.$$
$$\left. + e^{-1\%\times1} \times \max\left[0; S_0 \times e^{-(4\%-1\%)\times1} - S_{12M}\right]\right\}$$

$$= \frac{x\%}{S_0} \times \left\{c_{12M}\left(S_{12M};1.2S_0;24M\right) + c_{12M}^{d}\left(S_{12M};1.2S_0;24M; M = 0.2S_0\right)\right.$$
$$\left. + e^{-1\%\times1} \times p_{12M}\left(S_{12M}; S_0 \times e^{-(4\%-1\%)\times1};12M\right)\right\}$$

Consequentemente, o valor actual da remuneração variável é dado por:

$$RV_0 = \frac{x\%}{S_0} \times \left\{c_0\left(S_0;1.2S_0;24M\right) + c_0^{d}\left(S_0;1.2S_0;24M; M = 0.2S_0\right)\right.$$
$$\left. + e^{-1\%\times1} \times p_0\left(S_0; S_0 \times e^{-(4\%-1\%)\times1};12M\right)\right\}$$

$$= \frac{x\%}{7{,}205} \times \left\{c_0\left(S_0;8{,}646;24M\right) + c_0^{d}\left(S_0;8{,}646;24M; M = 1{,}441\right)\right.$$
$$\left. + e^{-1\%\times1} \times p_0\left(S_0;6{,}992.06;12M\right)\right\}.$$

Utilizando as cotações constantes no enunciado,

$$RV_0 = \frac{x\%}{7{,}205} \times \left[419.99 + c_0^{d}\left(S_0;8{,}646;24M; M = 1{,}441\right) + e^{-1\%\times1} \times 428.28\right].$$

Relativamente à *cash-or-nothing* call e utilizando o enunciado,

$$c_0^d\left(S_0;8,646;24M;M=1,441\right)=1,441\times 0.2445\cong 352.30.$$

Para que a emissão possa ser feita ao par e com uma margem de 1%,

$$100\%-1\%=100\%\times e^{-3\%\times 2}+\frac{x\%}{7,205}\times\left[419.99+352.30+e^{-1\%\times 1}\times 428.28\right]$$

$$\Leftrightarrow x\%\cong 29.05\%$$