Modelos de Estrutura Temporal de Taxas de Juro
Mestrado em Matemática Financeira 06/07
IBS e FCUL
Exame 1ª Época - Resolução

13/Dez/07 Duração: 3h

1. (a) Via Proposição 21 dos apontamentos de "Opções Exóticas":

$$AYLIS_t = c_t\left(S_t, X, T_2\right) + e^{-q(T_2-T_1)} p_t\left(S_t, Xe^{-(r-q)(T_2-T_1)}, T_1\right). \tag{1}$$

Por outro lado, o processo

$$dS_t = (r-q)\, S_t dt + \delta\sqrt{S_t} dW_t^{\mathbb{Q}}$$

é um processo CEV com

$$\beta = \frac{1}{2}\,(<2). \tag{2}$$

Consequentemente, a call e a put Europeias contidas na equação (1) podem ser avaliadas via Proposição 14 dos apontamentos de METTJ, sujeito à condição (2).

(b) O preço, no momento actual t, do futuro é dado por:

$$\begin{aligned} F\left(t, T_f, T_1\right) &= \mathbb{E}_{\mathbb{Q}}\left[\left. F\left(T_f, T_f, T_1\right)\right| \mathcal{F}_t\right] \\ &= \mathbb{E}_{\mathbb{Q}}\left[\left. P\left(T_f, T_1\right)\right| \mathcal{F}_t\right]. \end{aligned}$$

Considerando que, no modelo CIR, o preço da obrigação de cupão zero subjacente é dado pela Proposição 64, então:

$$\begin{aligned} F\left(t, T_f, T_1\right) &= \mathbb{E}_{\mathbb{Q}}\left\{\left.\exp\left[A\left(T_1-T_f\right) + B\left(T_1-T_f\right) r_{T_f}\right]\right| \mathcal{F}_t\right\} \\ &= \exp\left[A\left(T_1-T_f\right)\right] \mathbb{E}_{\mathbb{Q}}\left\{\left.\exp\left[B\left(T_1-T_f\right) r_{T_f}\right]\right| \mathcal{F}_t\right\}. \end{aligned} \tag{3}$$

Finalmente, o valor esperado contido na equação (3) pode ser interpretado como correspondendo à transformada de Laplace da taxa de juro instantânea e calculado, portanto, via Proposição 66 dos apontamentos com $\lambda = -B\left(T_1-T_f\right)$ e $\mu = 0$:

$$F\left(t, T_f, T_1\right) = \exp\left[A\left(T_1-T_f\right) + \phi_{-B\left(T_1-T_f\right),0}\left(T_f-t\right) - r_t\psi_{-B\left(T_1-T_f\right),0}\left(T_f-t\right)\right].$$

2. (a) Utilizando a Proposição 22 dos apontamentos,

$$\begin{aligned} p_0 = &-100 \times e^{-1\%\times0.5} \times \left[1 - P_1\left(S_t = 100, v_t = 0.02; T = 0.5, X = 100\right)\right] \\ &+ e^{-4\%\times0.5} \times 100 \times \left[1 - P_2\left(S_t = 100, v_t = 0.02; T = 0.5, X = 100\right)\right]. \end{aligned} \tag{4}$$

Com base nas equações (173) e (174) dos apontamentos:

$$
\begin{aligned}
P_1\left(S_t=100, v_t=0.02; T=0.5, X=100\right) &\approx \frac{1}{2}+\frac{0.30001240}{\pi} \\
&\cong 5.9550E-01, \qquad (5)
\end{aligned}
$$

e

$$
\begin{aligned}
P_2\left(S_t=100, v_t=0.02; T=0.5, X=100\right) &\approx \frac{1}{2}+\frac{0.16593459}{\pi} \\
&\cong 5.5282E-01. \qquad (6)
\end{aligned}
$$

Combinando as equações (4), (5) e (6),

$$
\begin{aligned}
p_0 &= -100\times e^{-1\%\times 0.5}\times\left(1-5.9550E-01\right)+e^{-4\%\times 0.5}\times 100\times\left(1-5.5282E-01\right) \\
&\cong EUR3.58410.
\end{aligned}
$$

(b) O payoff terminal da Asset-or-Nothing put é dado por:

$$
p_T^A = MS_T\mathbb{1}_{\{S_T<X\}},
$$

onde $M=1$, $T=0.5$ e $X=EUR100$. Portanto, o valor actual da Asset-or-Nothing put é igual a:

$$
\begin{aligned}
p_t^A &= e^{-r(T-t)}M\mathbb{E}_{\mathbb{Q}}\left(S_T\mathbb{1}_{\{S_T<X\}}\middle|\mathcal{F}_t\right) \\
&= MS_te^{qt}\mathbb{E}_{\mathbb{Q}^S}\left(\frac{S_T\mathbb{1}_{\{S_T<X\}}}{S_Te^{qT}}\middle|\mathcal{F}_t\right) \\
&= MS_te^{-q(T-t)}\mathbb{E}_{\mathbb{Q}^S}\left(\mathbb{1}_{\{S_T<X\}}\middle|\mathcal{F}_t\right) \\
&= MS_te^{-q(T-t)}\mathbb{Q}^S\left(S_T<X\middle|\mathcal{F}_t\right) \\
&= MS_te^{-q(T-t)}\left[1-P_1\left(S_t, v_t; T, X\right)\right]. \qquad (7)
\end{aligned}
$$

Combinando as equações (5) e (7),

$$
\begin{aligned}
p_t^A &= EUR100\times e^{-1\%\times 0.5}\times\left(1-5.9550E-01\right) \\
&\cong EUR40.249.
\end{aligned}
$$

3. (a) Via equações (269) e (270) dos apontamentos,

$$
\begin{aligned}
B(4) &= \frac{1-e^{-0.4\times 4}}{0.4} \\
&\cong 3.6964,
\end{aligned}
$$

e

$$
\begin{aligned}
A(4) &= (3.6964-4)\left(4\%-\frac{0.05^2}{2\,(0.4)^2}\right)-\frac{0.05^2}{4\times 0.4}\times(3.6964)^2 \\
&\cong -0.1729.
\end{aligned}
$$

Portanto,

$$
\begin{aligned}
P(0,4) &= \exp\left(-0.1729-3.6964\times 4.5\%\right) \\
&\cong 0.7123.
\end{aligned}
$$

(b) Utilizando a Proposição 59 dos apontamentos e uma vez que

$$\begin{aligned} K &= P(0,1,3) \\ &= \frac{P(0,3)}{P(0,1)} \\ &= \frac{0.7958}{0.9453} \\ &\cong 0.8419, \end{aligned}$$

então

$$\begin{aligned} & p_0 \left[P(0,3); 84.19\%; 1 \right] \\ = \;& -P(0,3)\, \Phi\left(-d_1^V\right) + 84.19\% \times P(0,1)\, \Phi\left(-d_0^V\right) \\ = \;& -0.7958 \times \Phi\left(-d_1^V\right) + 84.19\% \times 0.9453 \times \Phi\left(-d_0^V\right), \end{aligned}$$

onde

$$\begin{aligned} v(0,1,3) &= \sqrt{\frac{0.05^2}{0.4^2}\left[1 - e^{-0.4(3-1)}\right]^2 \frac{1 - e^{-2\times 0.4\times 1}}{2\times 0.4}} \\ &\cong 5.711\%, \end{aligned}$$

$$\begin{aligned} d_1^V &= \frac{\ln\left[\frac{0.7958}{84.19\% \times 0.9453}\right] + \frac{(5.711\%)^2}{2}}{5.711\%} \\ &\cong 0.028554422, \end{aligned}$$

e

$$\begin{aligned} d_0^V &= 0.028554422 - 5.711\% \\ &\cong -0.028554422. \end{aligned}$$

Portanto,

$$\begin{aligned} & p_0 \left[P(0,3); 84.19\%; 1 \right] \\ = \;& -0.7958 \times \Phi(-0.028554422) + 84.19\% \times 0.9453 \times \Phi(0.028554422) \\ = \;& -0.7958 \times 0.488609982 + 84.19\% \times 0.9453 \times 0.511390018 \\ \cong \;& 1.813\%. \end{aligned}$$

(c) O payoff terminal do contrato em análise ocorre daqui a 3 anos e é igual a

$$V_3 = EUR100,000 \times \left[1 + 6 \times \frac{E(2.5,3)}{2} \times \mathbb{1}_{\{E(2.5,3) < 5\%\}}\right],$$

onde $E(2.5,3)$ designa o valor da Euribor a 6 meses em vigor daqui a 2.5 anos. Em termos genéricos,

$$V_{T+\delta} = M \left[1 + nE(T, T+\delta)\, \delta \mathbb{1}_{\{E(T,T+\delta) < E_l\}}\right],$$

onde $M \equiv EUR100,000$, $n \equiv 6$, $T \equiv 2.5$, $\delta \equiv 0.5$ e $E_l \equiv 5\%$.
Trabalhando na forward measure $\mathbb{Q}_{T+\delta}$ associada ao numerário PDB com vencimento no momento $(T+\delta)$,

$$V_t = MP(t,T+\delta) + nMP(t,T+\delta)\,\mathbb{E}_{\mathbb{Q}_{T+\delta}}\left[\left.\frac{E(T,T+\delta)\,\delta\mathbb{1}_{\{E(T,T+\delta)<E_l\}}}{P(T+\delta,T+\delta)}\right|\mathcal{F}_t\right]. \tag{8}$$

Uma vez que

$$E(T,T+\delta) = \frac{1}{\delta}\left[\frac{1}{P(T,T+\delta)} - 1\right]$$

e $P(T+\delta,T+\delta)=1$, então

$$\begin{aligned}
& nMP(t,T+\delta)\,\mathbb{E}_{\mathbb{Q}_{T+\delta}}\left[\left.\frac{E(T,T+\delta)\,\delta\mathbb{1}_{\{E(T,T+\delta)<E_l\}}}{P(T+\delta,T+\delta)}\right|\mathcal{F}_t\right] \\
= \; & nMP(t,T+\delta)\,\mathbb{E}_{\mathbb{Q}_{T+\delta}}\left[\left.P(T,T+\delta)^{-1}\,\mathbb{1}_{\left\{P(T,T+\delta)^{-1}<1+\delta E_l\right\}}\right|\mathcal{F}_t\right] \\
& -nMP(t,T+\delta)\,\mathbb{Q}_{T+\delta}\left[\left.P(T,T+\delta)^{-1}<1+\delta E_l\right|\mathcal{F}_t\right] \\
= \; & nMP(t,T+\delta)\,\mathbb{E}_{\mathbb{Q}_{T+\delta}}\left[\left.P(T,T,T+\delta)^{-1}\,\mathbb{1}_{\left\{P(T,T,T+\delta)^{-1}<1+\delta E_l\right\}}\right|\mathcal{F}_t\right] \\
& -nMP(t,T+\delta)\,\mathbb{Q}_{T+\delta}\left[\left.P(T,T,T+\delta)^{-1}<1+\delta E_l\right|\mathcal{F}_t\right].
\end{aligned} \tag{9}$$

A equação (321) dos apontamentos implica que

$$P(T,T,T+\delta)^{-1} = P(t,T,T+\delta)^{-1}\exp\left[-\frac{1}{2}v^2(t,T,T+\delta) + Y^{\mathbb{Q}_{T+\delta}}\right], \tag{10}$$

onde

$$Y^{\mathbb{Q}_{T+\delta}} \overset{\mathbb{Q}_{T+\delta}}{\sim} N^1\left(0, v^2(t,T,T+\delta)\right). \tag{11}$$

Consequentemente,

$$\begin{aligned}
& \mathbb{Q}_{T+\delta}\left[\left.P(T,T,T+\delta)^{-1}<1+\delta E_l\right|\mathcal{F}_t\right] \\
= \; & \mathbb{Q}_{T+\delta}\left\{\left.P(t,T,T+\delta)^{-1}\exp\left[-\frac{1}{2}v^2(t,T,T+\delta)+Y^{\mathbb{Q}_{T+\delta}}\right]<(1+\delta E_l)\right|\mathcal{F}_t\right\} \\
= \; & \mathbb{Q}_{T+\delta}\left\{\left.Y^{\mathbb{Q}_{T+\delta}} < \ln\left[(1+\delta E_l)\,P(t,T,T+\delta)\right] + \frac{1}{2}v^2(t,T,T+\delta)\right|\mathcal{F}_t\right\} \\
= \; & \Phi\left\{\frac{\ln\left[(1+\delta E_l)\,P(t,T,T+\delta)\right]+\frac{1}{2}v^2(t,T,T+\delta)}{v(t,T,T+\delta)}\right\}.
\end{aligned} \tag{12}$$

Por outro lado,

$$
\begin{aligned}
& \mathbb{E}_{\mathbb{Q}_{T+\delta}}\left[P(T,T,T+\delta)^{-1}\mathbb{1}_{\left\{P(T,T,T+\delta)^{-1}<1+\delta E_l\right\}}\middle|\mathcal{F}_t\right] \\
= & \int_{-\infty}^{\ln[(1+\delta E_l)P(t,T,T+\delta)]+\frac{1}{2}v^2(t,T,T+\delta)} P(t,T,T+\delta)^{-1}\exp\left[-\frac{1}{2}v^2(t,T,T+\delta)+z\right] \\
& \frac{\exp\left[-\frac{1}{2}\frac{z^2}{v^2(t,T,T+\delta)}\right]}{\sqrt{2\pi v^2(t,T,T+\delta)}}dz \\
= & P(t,T,T+\delta)^{-1}\int_{-\infty}^{\ln[(1+\delta E_l)P(t,T,T+\delta)]+\frac{1}{2}v^2(t,T,T+\delta)} \\
& \frac{\exp\left[-\frac{1}{2}\frac{z^2-2v^2(t,T,T+\delta)z+v^4(t,T,T+\delta)}{v^2(t,T,T+\delta)}\right]}{\sqrt{2\pi v^2(t,T,T+\delta)}}dz \\
= & P(t,T,T+\delta)^{-1}\int_{-\infty}^{1+\delta E_l}\frac{\exp\left\{-\frac{1}{2}\frac{\left[z-v^2(t,T,T+\delta)\right]^2}{v^2(t,T,T+\delta)}\right\}}{\sqrt{2\pi v^2(t,T,T+\delta)}}dz \\
= & P(t,T,T+\delta)^{-1} \qquad (13) \\
& \Phi\left\{\frac{\ln[(1+\delta E_l)P(t,T,T+\delta)]+\frac{1}{2}v^2(t,T,T+\delta)-v^2(t,T,T+\delta)}{v(t,T,T+\delta)}\right\}.
\end{aligned}
$$

Em suma, combinando as equações (8), (9), (12) e (13), o valor actual do contrato é dado por:

$$
\begin{aligned}
V_t = & MP(t,T+\delta) \qquad (14) \\
& +nMP(t,T)\Phi\left\{\frac{\ln[(1+\delta E_l)P(t,T,T+\delta)]-\frac{1}{2}v^2(t,T,T+\delta)}{v(t,T,T+\delta)}\right\} \\
& -nMP(t,T+\delta)\Phi\left\{\frac{\ln[(1+\delta E_l)P(t,T,T+\delta)]+\frac{1}{2}v^2(t,T,T+\delta)}{v(t,T,T+\delta)}\right\}.
\end{aligned}
$$

Adaptando a equação (14) ao exemplo em apreço,

$$
\begin{aligned}
V_0 = & EUR100,000\times P(0,3) \\
& +6\times EUR100,000\times P(0,2.5)\times\Phi\left\{\frac{\ln\left[\left(1+\frac{5\%}{2}\right)\frac{P(0,3)}{P(0,2.5)}\right]-\frac{1}{2}v^2(0,2.5,3)}{v(0,2.5,3)}\right\} \\
& -6\times EUR100,000\times P(0,3)\times\Phi\left\{\frac{\ln\left[\left(1+\frac{5\%}{2}\right)\frac{P(0,3)}{P(0,2.5)}\right]+\frac{1}{2}v^2(0,2.5,3)}{v(0,2.5,3)}\right\}.
\end{aligned}
$$

Visto que

$$
\begin{aligned}
v(0,1,3) & = \sqrt{\frac{0.05^2}{0.4^2}\left[1-e^{-0.4(3-2.5)}\right]^2\frac{1-e^{-2\times 0.4\times 2.5}}{2\times 0.4}} \\
& \cong 2.356\%,
\end{aligned}
$$

então

$$
\begin{aligned}
V_0 &= EUR100,000 \times 0.7958 \\
&+6 \times EUR100,000 \times 0.8364 \times \Phi \left\{ \frac{\ln \left[\left(1+\frac{5\%}{2}\right) \frac{0.7958}{0.8364} \right] - \frac{1}{2} \left(2.356\%\right)^2}{2.356\%} \right\} \\
&-6 \times EUR100,000 \times 0.7958 \times \Phi \left\{ \frac{\ln \left[\left(1+\frac{5\%}{2}\right) \frac{0.7958}{0.8364} \right] + \frac{1}{2} \left(2.356\%\right)^2}{2.356\%} \right\},
\end{aligned}
$$

i.e.

$$
\begin{aligned}
V_0 &= EUR100,000 \times 0.7958 \\
&+6 \times EUR100,000 \times 0.8364 \times 0.141839986 \\
&-6 \times EUR100,000 \times 0.7958 \times 0.147196652 \\
&\cong EUR80,474.86.
\end{aligned}
$$

4. (a) Via equação (375) dos apontamentos,

$$
\begin{aligned}
c_0 \left[P\left(0,3\right);88.997\%;1\right] &= P\left(0,3\right) F_{\chi^2_{\left(\frac{4\times 0.6\times 6\%}{0.05^2},\zeta_2\right)}} \left(\frac{r^*}{L_2}\right) \\
&\quad -88.997\% \times P\left(0,1\right) F_{\chi^2_{(57.6,\zeta_1)}} \left(\frac{r^*}{L_1}\right),
\end{aligned}
\tag{15}
$$

sendo

$$
\begin{aligned}
\gamma &= \sqrt{0.6^2 + 2 \times \left(5\%\right)^2} \\
&\cong 0.604152299,
\end{aligned}
$$

$$
\begin{aligned}
\zeta_2 &= \frac{8 r_t \gamma^2 e^{\gamma(T_1 - t)}}{\sigma^2 \left[e^{\gamma(T_1-t)} - 1\right] \left\{\gamma \left[e^{\gamma(T_1-t)} + 1\right] + \left[k - \sigma^2 B\left(T_2 - T_1\right)\right] \left[e^{\gamma(T_1-t)} - 1\right]\right\}} \\
&= \left[8 \times 4\% \times \left(0.604152299\right)^2 \times e^{0.604152299 \times 1}\right] \Big\{0.05^2 \times \left(e^{0.604152299\times 1} - 1\right) \\
&\quad 0.604152299 \times \left(e^{0.604152299\times 1} + 1\right) \\
&\quad + \left(0.6 - 0.05^2 \times \left(-1.1636\right)\right) \left(e^{0.604152299\times 1} - 1\right)\Big] \Big\}^{-1} \\
&\cong 46.62380737,
\end{aligned}
$$

$$
\begin{aligned}
L_2 &= \frac{\sigma^2}{2} \frac{e^{\gamma(T_1-t)} - 1}{\gamma \left[e^{\gamma(T_1-t)} + 1\right] + \left[k - \sigma^2 B\left(T_2 - T_1\right)\right] \left[e^{\gamma(T_1-t)} - 1\right]} \\
&= \frac{\frac{0.05^2}{2} \times \left(e^{0.604152299} - 1\right)}{0.604152299 \times \left[e^{0.604152299} + 1\right] + \left[0.6 - 0.05^2 \times \left(-1.1636\right)\right] \left(e^{0.604152299} - 1\right)} \\
&\cong 0.000469329
\end{aligned}
$$

e

$$
\begin{aligned}
r^* &= \frac{\ln(K) - A(T_2 - T_1)}{B(T_2 - T_1)} \\
&= \frac{\ln(88.997\%) - A(3-1)}{B(2)} \\
&= \frac{\ln(94.018\%) - (-0.0501)}{-1.1636} \\
&\cong 5.712\%,
\end{aligned}
$$

Portanto,

$$
\begin{aligned}
c_0\left[P(0,3); 88.997\%; 1\right] &= 0.8590 \times F_{\chi^2_{(57.6,46.62380737)}}\left(\frac{5.712\%}{0.000469329}\right) \\
&\quad -88.997\% \times 0.9560 \times F_{\chi^2_{(57.6,46.6747866)}}\left(\frac{5.712\%}{0.000470}\right).
\end{aligned} \tag{16}
$$

$$
\begin{aligned}
P(0,3) &= \exp\left[-0.1000 - 0.4982 \times 5\%\right] \\
&\cong 0.8826.
\end{aligned}
$$

As probabilidades contidas na equação anterior podem ser calculada via aproximação de Sankaran, i.e.

$$
\begin{aligned}
F_{\chi^2(a,b)}(z) &= \mathbb{P}\left(\chi^2(a,b) \leq z\right) \\
&= \mathbb{P}\left\{\left[\frac{\chi^2(a,b)}{a+b}\right]^h \leq \left(\frac{z}{a+b}\right)^h\right\} \\
&\approx \Phi\left[\frac{\left(\frac{z}{a+b}\right)^h - \mu_h}{\sigma_h}\right],
\end{aligned} \tag{17}
$$

onde

$$
\mu_h := 1 + h(h-1)\frac{a+2b}{(a+b)^2} - h(h-1)(2-h)(1-3h)\frac{(a+2b)^2}{2(a+b)^4}, \tag{18}
$$

$$
\sigma_h^2 := h^2 \frac{2(a+2b)}{(a+b)^2}\left[1 - (1-h)(1-3h)\frac{a+2b}{(a+b)^2}\right], \tag{19}
$$

e

$$
h := 1 - \frac{2}{3}(a+b)(a+3b)(a+2b)^{-2}. \tag{20}
$$

Começando por $F_{\chi^2_{(57.6,46.62380737)}}\left(\frac{5.712\%}{0.000469329}\right)$, $a = 57.6$, $b = 46.62380737$,

$$
\begin{aligned}
h &= 1 - \frac{2}{3}(57.6 + 46.62380737)(57.6 + 3 \times 46.62380737) \\
&\quad (57.6 + 2 \times 46.62380737)^{-2} \\
&\cong 0.397019824,
\end{aligned}
$$

$$
\begin{aligned}
\mu_h &= 1 + 0.397019824 \times (0.397019824 - 1) \frac{57.6 + 2 \times 46.62380737}{(57.6 + 46.62380737)^2} \\
&\quad -0.397019824 \times (0.397019824 - 1)(2 - 0.397019824) \\
&\quad (1 - 3 \times 0.397019824) \frac{(57.6 + 2 \times 46.62380737)^2}{2(57.6 + 46.62380737)^4} \\
&\cong 0.99666848,
\end{aligned}
$$

e

$$
\begin{aligned}
\sigma_h^2 &:= 0.397019824^2 \times \frac{2(57.6 + 2 \times 46.62380737)}{(57.6 + 46.62380737)^2} \\
&\quad \left[1 - (1 - 0.397019824)(1 - 3 \times 0.397019824) \frac{57.6 + 2 \times 46.62380737}{(57.6 + 46.62380737)^2}\right] \\
&\cong 0.004384834.
\end{aligned}
$$

Utilizando a equação (17),

$$
\begin{aligned}
F_{\chi^2_{(57.6,46.62380737)}}\left(\frac{5.712\%}{0.000469329}\right) &= \Phi\left[\frac{\left(\frac{121.714681}{57.6+46.62380737}\right)^{0.397019824} - 0.99666848}{\sqrt{0.004384834}}\right] \\
&\cong 0.843683429. \qquad (21)
\end{aligned}
$$

Finalmente, combinando as equações (16) e (21),

$$
\begin{aligned}
c_0[P(0,3); 88.997\%; 1] &= 0.8590 \times 0.843683429 \\
&\quad -88.997\% \times 0.9560 \times 0.841287624 \\
&\cong 0.00889560. \qquad (22)
\end{aligned}
$$

(b) De acordo com a Proposição 61 dos apontamentos, o valor actual da *call* sobre a CBB pode ser decomposto numa carteira de 2 *calls* Europeias sobre PBD:

$$
\begin{aligned}
& c_0(B_t; X = 100\%; T = 1) \\
= \; & 6\% \times c_0[P(0,2); X_1; T = 1] + 106\% \times c_0[P(0,3); X_2; T = 1].
\end{aligned}
$$

Os *strikes* podem ser obtidos via equação (327) dos apontamentos, uma vez adaptada para o modelo CIR via Proposição 64:

$$
\begin{aligned}
X_1 &= \exp[A(2-1) + B(2-1) \times 5.712\%] \\
&= \exp(-0.0149 - 0.7517 \times 5.712\%) \\
&\cong 94.382\%,
\end{aligned}
$$

e

$$
\begin{aligned}
X_2 &= \exp[A(3-1) + B(3-1) \times 5.712\%] \\
&= \exp(-0.0501 - 1.1636 \times 5.712\%) \\
&\cong 88.997\%.
\end{aligned}
$$

Portanto,

$$\begin{aligned} & c_0\left(B_t; X = 100\%; T = 1\right) \\ = \; & 6\% \times c_0\left[P\left(0,2\right); 94.382\%; T = 1\right] + 106\% \times c_0\left[P\left(0,3\right); 88.997\%; T = 1\right]. \end{aligned} \tag{23}$$

A segunda *call* já foi avaliada na alínea anterior –vide equação (22):

$$c_0\left[P\left(0,3\right); 88.997\%; T = 1\right] \cong 0.889560\%. \tag{24}$$

Relativamente à primeira *call*, o enunciado fornece o seguinte valor actual:

$$c_0\left[P\left(0,2\right); 94.382\%; T = 1\right] = 0.608\%. \tag{25}$$

Combinando as equações (23), (24) e (25),

$$\begin{aligned} & c_0\left(B_t; X = 100\%; T = 1\right) \\ = \; & 6\% \times 0.608\% + 106\% \times 0.889560\% \\ \cong \; & 0.979\%. \end{aligned}$$

Referências