# Modelos de Estrutura Temporal de Taxas de Juro
# Mestrado em Matemática Financeira 09/10
# IBS e FCUL
## Exame 1ª Época - Resolução

07/Dez/10 Duração: 3h

1. (a) Utilizando a solução dada pelo enunciado para $v_T$, então

$$\begin{aligned}
& \mathbb{E}_{\mathbb{Q}}\left[\exp\left(\omega v_T\right)|x_T=\bar{x},v_t\right] \\
= \; & \exp\left\{\omega v_t+\omega k\theta\left(T-t\right)+\omega\sigma\rho\left[\bar{x}-x_t-\left(r-q\right)\left(T-t\right)\right]\right\} \\
& \mathbb{E}_{\mathbb{Q}}\left\{\exp\left[\omega\left(\frac{\sigma\rho}{2}-k\right)\int_t^T v_u du+\omega\sigma\sqrt{1-\rho^2}\int_t^T\sqrt{v_u}dZ_2^{\mathbb{Q}}\left(u\right)\right]|v_t\right\}. \quad (1)
\end{aligned}$$

A questão reside, portanto, em calcular o valor esperado contido no lado direito da equação (1).

Tal valor esperado pode ser reescrito da seguinte forma:

$$\begin{aligned}
& \mathbb{E}_{\mathbb{Q}}\left\{\exp\left[\omega\left(\frac{\sigma\rho}{2}-k\right)\int_t^T v_u du+\omega\sigma\sqrt{1-\rho^2}\int_t^T\sqrt{v_u}dZ_2^{\mathbb{Q}}\left(u\right)\right]|v_t\right\} \quad (2) \\
= \; & \mathbb{E}_{\mathbb{Q}}\left\{\exp\left[\omega\left(\frac{\sigma\rho}{2}-k+\frac{\omega\sigma^2\left(1-\rho^2\right)}{2}\right)\int_t^T v_u du\right.\right. \\
& \left.\left.+\omega\sigma\sqrt{1-\rho^2}\int_t^T\sqrt{v_u}dZ_2^{\mathbb{Q}}\left(u\right)-\frac{\omega^2\sigma^2\left(1-\rho^2\right)}{2}\int_t^T v_u du|v_t\right]\right\}
\end{aligned}$$

Utilizando a tower law,

$$\begin{aligned}
& \mathbb{E}_{\mathbb{Q}}\left\{\exp\left[\omega\left(\frac{\sigma\rho}{2}-k\right)\int_t^T v_u du+\omega\sigma\sqrt{1-\rho^2}\int_t^T\sqrt{v_u}dZ_2^{\mathbb{Q}}\left(u\right)\right]|v_t\right\} \quad (3) \\
= \; & \mathbb{E}_{\mathbb{Q}}\left\{\exp\left[\omega\left(\frac{\sigma\rho}{2}-k+\frac{\omega\sigma^2\left(1-\rho^2\right)}{2}\right)\int_t^T v_u du\right]\right. \\
& \left.\mathbb{E}_{\mathbb{Q}}\left[\exp\left(\omega\sigma\sqrt{1-\rho^2}\int_t^T\sqrt{v_u}dZ_2^{\mathbb{Q}}\left(u\right)-\frac{\omega^2\sigma^2\left(1-\rho^2\right)}{2}\int_t^T v_u du\right)\middle|\int_t^T v_u du\right]|v_t\right\}.
\end{aligned}$$

Via Shreve (2004, equação 4.4.31), sabemos que

$$\mathbb{E}_{\mathbb{Q}}\left[\exp\left(\alpha\int_t^T g\left(u\right)dW\left(u\right)-\frac{\alpha}{2}\int_t^T g^2\left(u\right)du\right)\right]=1, \quad (4)$$

para $\alpha\in\mathbb{R}$ e sendo $g\left(u\right)$ uma qualquer função aleatória e $W$ um Brownian motion standard. Combinando as equações (3) e (4), então:

$$\begin{aligned}
& \mathbb{E}_{\mathbb{Q}}\left\{\exp\left[\omega\left(\frac{\sigma\rho}{2}-k\right)\int_t^T v_u du+\omega\sigma\sqrt{1-\rho^2}\int_t^T\sqrt{v_u}dZ_2^{\mathbb{Q}}\left(u\right)\right]|v_t\right\} \quad (5) \\
= \; & \mathbb{E}_{\mathbb{Q}}\left\{\exp\left[\omega\left(\frac{\sigma\rho}{2}-k+\frac{\omega\sigma^2\left(1-\rho^2\right)}{2}\right)\int_t^T v_u du\right]\middle|v_t\right\}.
\end{aligned}$$

O lado direito da equação (5) não é mais do que a função geradora de momentos da varável aleatória $\int_t^T v_u$, a qual pode ser obtida via Proposição 66 dos apontamentos já que no modelo de Heston $v_t$ segue um square-root process:

$$\begin{aligned} & \mathbb{E}_{\mathbb{Q}}\left\{\exp\left[\omega\left(\frac{\sigma\rho}{2}-k+\frac{\omega\sigma^2\left(1-\rho^2\right)}{2}\right)\int_t^T v_u du\right]\middle| v_t\right\} \qquad (6) \\ = & \exp\left[\phi_{0,-\beta}\left(T-t\right)-r_t\psi_{0,-\beta}\left(T-t\right)\right], \end{aligned}$$

sendo

$$\beta := \omega\left(\frac{\sigma\rho}{2}-k+\frac{\omega\sigma^2\left(1-\rho^2\right)}{2}\right).$$

(b) O payoff final de de uma *cash-or-nothing call* com vencimento no momento $T+\delta$, com um *contract size* igual a $M$, com um *strike* igual a $k$, e sobre a Euribor $E\left(T,T+\delta\right)$ a vigorar entre os momentos $T$ $(\geq t)$ e $T+\delta$ (com $\delta>0$) é dado por:

$$V_{T+\delta} = M\mathbb{1}_{\{E(T,T+\delta)>k\}}. \qquad (7)$$

Utilizando a identidade

$$P\left(T,T+\delta\right) = \left[1+\delta\times E\left(T,T+\delta\right)\right]^{-1},$$

a equação (7) pode ser reescrita como

$$\begin{aligned} V_{T+\delta} &= M\mathbb{1}_{\{P^{-1}(T,T+\delta)-1>k\delta\}} \\ &= M\mathbb{1}_{\{P^{-1}(T,T+\delta)>1+k\delta\}} \\ &= M\mathbb{1}_{\left\{P(T,T,T+\delta)>(1+k\delta)^{-1}\right\}}. \end{aligned}$$

Consequentemente e utilizando a *forward measure* $\mathbb{Q}_{T+\delta}$ associada ao numerário $P\left(t,T+\delta\right)$, então

$$\begin{aligned} V_t &= P\left(t,T+\delta\right)\mathbb{E}_{\mathbb{Q}_{T+\delta}}\left(\frac{M\mathbb{1}_{\left\{P(T,T,T+\delta)>(1+k\delta)^{-1}\right\}}}{P\left(T+\delta,T+\delta\right)}\middle|\mathcal{F}_t\right) \\ &= MP\left(t,T+\delta\right)\mathbb{Q}_{T+\delta}\left[P\left(T,T,T+\delta\right)>\left(1+k\delta\right)^{-1}\middle|\mathcal{F}_t\right]. \qquad (8) \end{aligned}$$

Via equação (321) dos apontamentos, sabemos que

$$P\left(T,T,T+\delta\right) = P\left(t,T,T+\delta\right)\exp\left[\frac{1}{2}v^2\left(t,T,T+\delta\right)-Y^{\mathbb{Q}_{T+\delta}}\right], \qquad (9)$$

onde

$$Y^{\mathbb{Q}_{T+\delta}}\middle|\mathcal{F}_t \sim N^1\left(0,v^2\left(t,T,T+\delta\right)\right). \qquad (10)$$

Combinando as equações (8), (9) e (10),

$$\begin{aligned}
& V_t \\
= \; & MP(t,T+\delta)\,\mathbb{Q}_{T+\delta}\left\{ P(t,T,T+\delta)\exp\left[\frac{1}{2}v^2(t,T,T+\delta)-Y^{\mathbb{Q}_{T+\delta}}\right] > (1+k\delta)^{-1}\middle|\, \mathcal{F}_t\right\} \\
= \; & MP(t,T+\delta)\,\mathbb{Q}_{T+\delta}\left\{ -Y^{\mathbb{Q}_{T+\delta}} > \ln\left[\frac{(1+k\delta)^{-1}}{P(t,T,T+\delta)}\right]-\frac{1}{2}v^2(t,T,T+\delta)\middle|\, \mathcal{F}_t\right\} \\
= \; & MP(t,T+\delta)\,\mathbb{Q}_{T+\delta}\left\{ Y^{\mathbb{Q}_{T+\delta}} < \ln\left[\frac{P(t,T,T+\delta)}{(1+k\delta)^{-1}}\right]+\frac{1}{2}v^2(t,T,T+\delta)\middle|\, \mathcal{F}_t\right\} \\
= \; & MP(t,T+\delta)\,\Phi\left\{\frac{\ln\left[\frac{P(t,T,T+\delta)}{(1+k\delta)^{-1}}\right]+\frac{1}{2}v^2(t,T,T+\delta)}{v(t,T,T+\delta)}\right\}.
\end{aligned}$$

(c) Via Proposição 66 dos apontamentos, sabemos que a função geradora de momentos (com parâmetro $\omega$) da variável aleatória

$$y_T := \int_t^T r_s ds$$

é dada por

$$\begin{aligned}
M(\omega) &= \mathbb{E}_{\mathbb{Q}}\left[\exp(\omega y_T)\,|\mathcal{F}_t\right] \\
&= \exp\left[\phi_{0,-\omega}(T-t)-r_t\psi_{0,-\omega}(T-t)\right]. \qquad (11)
\end{aligned}$$

Por outro lado, também sabemos que

$$VAR(y_T|\mathcal{F}_t) = \mathbb{E}_{\mathbb{Q}}\left(y_T^2|\mathcal{F}_t\right)-\left[\mathbb{E}_{\mathbb{Q}}(y_T|\mathcal{F}_t)\right]^2. \qquad (12)$$

Ora, via diferenciação da função geradora de momentos (11) podemos obter os momentos contidos no lado direito da equação (12):

$$\begin{aligned}
\mathbb{E}_{\mathbb{Q}}(y_T|\mathcal{F}_t) &= \left.\frac{\partial M(\omega)}{\partial\omega}\right|_{\omega=0} \\
&= \left[\left.\frac{\partial\phi_{0,-\omega}(T-t)}{\partial\omega}\right|_{\omega=0} - r_t\left.\frac{\partial\psi_{0,-\omega}(T-t)}{\partial\omega}\right|_{\omega=0}\right], \qquad (13)
\end{aligned}$$

e

$$\begin{aligned}
\mathbb{E}_{\mathbb{Q}}\left(y_T^2|\mathcal{F}_t\right) &= \left.\frac{\partial^2 M(\omega)}{\partial\omega^2}\right|_{\omega=0} \\
&= \left[\left.\frac{\partial^2\phi_{0,-\omega}(T-t)}{\partial\omega^2}\right|_{\omega=0} - r_t\left.\frac{\partial^2\psi_{0,-\omega}(T-t)}{\partial\omega^2}\right|_{\omega=0}\right]. \qquad (14)
\end{aligned}$$

(a) Via equação (64) dos handouts, o valor da call Europeia (com $\beta > 2$) é dado por: sendo

$$c_t(S,X,T) = S_t e^{-q(T-t)} Q_{\chi^2\left(\frac{2}{\beta-2}, 2\kappa X^{2-\beta}\right)}(2x) - X e^{-r(T-t)} F_{\chi^2\left(2+\frac{2}{\beta-2}, 2x\right)}\left(2\kappa X^{2-\beta}\right), \tag{15}$$

$$\kappa := \frac{2(r-q)}{(2-\beta)\delta^2\left[e^{(2-\beta)(r-q)(T-t)} - 1\right]}, \tag{16}$$

e

$$x := \kappa S_t^{2-\beta} e^{(2-\beta)(r-q)(T-t)}. \tag{17}$$

Sendo a volatilidade da taxa de rentabilidade do activo subjacente é igual a 25% ao ano, então, e via equação (2) dos handouts,

$$\delta = \frac{25\%}{2^{\frac{4-2}{2}}} = 0.125.$$

Retomando as equações (16) e (17),

$$\kappa = \frac{2(1\% - 2\%)}{(2-4)(0.125)^2\left[e^{(2-4)(1\%-2\%)\times 1} - 1\right]} \cong 31.6811,$$

e

$$x = 31.6811 \times (2)^{2-4} e^{(2-4)(1\%-2\%)\times 1} \cong 8.08027.$$

Substituindo na equação (15), então

$$\begin{aligned} c_t &= 2 \times e^{-2\%\times 1} Q_{\chi^2\left(\frac{2}{4-2}, 2\times 31.6811\times(2)^{2-4}\right)}(2 \times 8.08027) \\ &\quad -2 \times e^{-1\%\times 1} F_{\chi^2\left(2+\frac{2}{4-2}, 2\times 8.08027\right)}\left(2 \times 31.6811 \times (2)^{2-4}\right) \\ &= 2 \times e^{-2\%\times 1} Q_{\chi^2(1,15.84053)}(16.16053) - 2 \times e^{-1\%\times 1} F_{\chi^2(3,16.16053)}(15.84053) \end{aligned} \tag{18}$$

Via tabela do enunciado, sabemos que

$$\begin{aligned} Q_{\chi^2(1,15.84053)}(16.16053) &= 1 - F_{\chi^2(1,15.84053)}(16.16053) \\ &= 1 - 0.516697341 \\ &= 0.483302659. \end{aligned} \tag{19}$$

A probabilidades $F_{\chi^2(3,16.16053)}(15.84053)$ pode ser calculada via aproximação de Sankaran, i.e.

$$\begin{aligned} F_{\chi^2(a,b)}(z) &= \mathbb{Q}\left(\chi^2(a,b) < z\right) \\ &= \mathbb{Q}\left\{\left[\frac{\chi^2(a,b)}{a+b}\right]^h < \left(\frac{z}{a+b}\right)^h\right\} \\ &\approx \Phi\left[\frac{\left(\frac{z}{a+b}\right)^h - \mu_h}{\sigma_h}\right], \end{aligned} \tag{20}$$

onde

$$\mu_h := 1 + h\,(h-1)\,\frac{a+2b}{(a+b)^2} - h\,(h-1)\,(2-h)\,(1-3h)\,\frac{(a+2b)^2}{2\,(a+b)^4}, \tag{21}$$

$$\sigma_h^2 := h^2 \frac{2\,(a+2b)}{(a+b)^2}\left[1-(1-h)\,(1-3h)\,\frac{a+2b}{(a+b)^2}\right], \tag{22}$$

e

$$h := 1 - \frac{2}{3}\,(a+b)\,(a+3b)\,(a+2b)^{-2}. \tag{23}$$

Visto que $a = 3$ e $b = 16.16053$, então

$$\begin{aligned} h &= 1 - \frac{2}{3}\,(3+16.16053)\,(3+3\times 16.16053) \\ &\quad (3+2\times 16.16053)^{-2} \\ &\cong 0.472890614, \end{aligned}$$

$$\begin{aligned} \mu_h &= 1 + 0.472890614 \times (0.472890614 - 1)\,\frac{3+2\times 16.16053}{(3+16.16053)^2} \\ &\quad -0.472890614 \times (0.472890614-1)\,(2-0.472890614) \\ &\quad (1-3\times 0.472890614)\,\frac{(3+2\times 16.16053)^2}{2\,(3+16.16053)^4} \\ &\cong 0.975280709, \end{aligned}$$

e

$$\sigma_h^2 := h^2 \frac{2\,(a+2b)}{(a+b)^2}\left[1-(1-h)\,(1-3h)\,\frac{a+2b}{(a+b)^2}\right]$$

$$\begin{aligned} \sigma_h^2 &= 0.472890614^2 \times \frac{2\,(3+2\times 16.16053)}{(3+16.16053)^2} \\ &\quad \left[1-(1-0.472890614)\,(1-3\times 0.472890614)\,\frac{3+2\times 16.16053}{(3+16.16053)^2}\right] \\ &\cong 0.043943482. \end{aligned}$$

Utilizando a equação (20),

$$\Phi\left[\frac{\left(\frac{z}{a+b}\right)^h - \mu_h}{\sigma_h}\right]$$

$$\begin{aligned} F_{\chi^2(3,16.16053)}\,(15.84053) &= \Phi\left[\frac{\left(\frac{15.84053}{3+16.16053}\right)^{0.472890614} - 0.975280709}{\sqrt{0.043943482}}\right] \\ &\cong 0.384921119. \end{aligned} \tag{24}$$

Finalmente, combinando as equações (18), (19) e (24),

$$\begin{aligned} c_t &= 2\times e^{-2\%\times 1} \times 0.483302659 - 2\times e^{-1\%\times 1}\times 0.384921119 \\ &\cong EUR0.18528. \end{aligned}$$

(b) O payoff terminal da cash-or-nothing put é dado por

$$CNP_T(S,X,T;M) = M\mathbb{1}_{\{S_T<X\}}.$$

Consequentemente,

$$\begin{aligned} CNP_t(S,X,T;M) &= e^{-r(T-t)}M\mathbb{E}_{\mathbb{Q}}\left(\mathbb{1}_{\{S_T<X\}}\middle|\mathcal{F}_t\right) \\ &= e^{-r(T-t)}M\mathbb{Q}\left(S_T<X\middle|\mathcal{F}_t\right) \\ &= e^{-r(T-t)}M\left[1-F_{\chi^2\left(2+\frac{2}{\beta-2},2x\right)}\left(2\kappa X^{2-\beta}\right)\right]. \end{aligned} \tag{25}$$

No caso em apreço,

$$\begin{aligned} CNP_t(S,X,T;M) &= EUR10\times e^{-1\%\times 1}\times(1-0.384921119) \\ &\cong EUR6.089587435. \end{aligned}$$

(a) Utilizando a Proposição 22 dos apontamentos,

$$\begin{aligned} p_0 = & -5\times e^{-0\%\times 0.5}\times\left[1-P_1\left(S_t=5,v_t=0.09;T=0.5,X=5\right)\right] \\ & +e^{-1\%\times 0.5}\times 5\times\left[1-P_2\left(S_t=5,v_t=0.09;T=0.5,X=5\right)\right]. \end{aligned} \tag{26}$$

Com base nas equações (173) e (174) dos apontamentos:

$$\begin{aligned} P_1\left(S_t=5,v_t=0.09;T=0.5,X=5\right) &\approx \frac{1}{2}+\frac{0.19711709}{\pi} \\ &\cong 5.6274E-01, \end{aligned} \tag{27}$$

e

$$\begin{aligned} P_2\left(S_t=5,v_t=0.09;T=0.5,X=5\right) &\approx \frac{1}{2}+\frac{-0.05169667}{\pi} \\ &\cong 4.8354E-01. \end{aligned} \tag{28}$$

Combinando as equações (26), (27) e (28),

$$\begin{aligned} p_0 &= -5\times e^{-0\%\times 0.5}\times(1-5.6274E-01)+e^{-1\%\times 0.5}\times 5\times(1-4.8354E-01) \\ &\cong EUR0.38312. \end{aligned}$$

(b) O payoff terminal de uma asset-or-nothing put é dado por

$$ANP_T(S,X,T) = MS_T\mathbb{1}_{\{S_T<X\}}.$$

Consequentemente,

$$\begin{aligned} ANP_t(S,X,T) &= e^{-r(T-t)}M\mathbb{E}_{\mathbb{Q}}\left(S_T\mathbb{1}_{\{S_T<X\}}\middle|\mathcal{F}_t\right) \\ &= S_te^{qt}M\mathbb{E}_{\mathbb{Q}_S}\left(\frac{S_T\mathbb{1}_{\{S_T<X\}}}{S_Te^{qT}}\middle|\mathcal{F}_t\right) \\ &= MS_te^{-q(T-t)}\mathbb{Q}_S\left(S_T<X\middle|\mathcal{F}_t\right). \end{aligned} \tag{29}$$

Visto que

$$\mathbb{Q}_S\left(S_T < X|\,\mathcal{F}_t\right) = 1 - P_1\left(S_t, v_t; T, X\right), \tag{30}$$

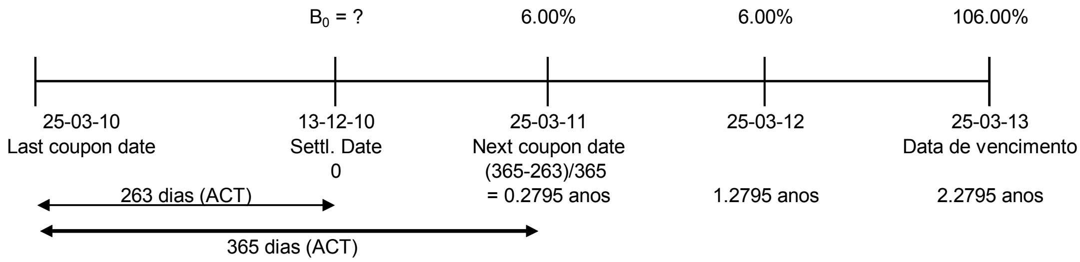
então combinando as equações (29) e (30),

$$ANP_t\left(S, X, T\right) = MS_t e^{-q(T-t)}\left[1 - P_1\left(S_t, v_t; T, X\right)\right]. \tag{31}$$

No caso em apreço

$$\begin{aligned} ANP_0 &= 1 \times 5 \times e^{-0\% \times 0.5} \times \left[1 - P_1\left(S_t = 5, v_t = 0.09; T = 0.5, X = 5\right)\right] \\ &= 1 \times 5 \times e^{-0\% \times 0.5} \times \left[1 - (5.6274E - 01)\right] \\ &\cong EUR2.186. \end{aligned}$$

(a) Settlement date = 08/12/10 + 5 dias de calendário = 13/12/10.
Pretende-se avaliar uma obrigação com os seguintes cash flows vincendos:




Portanto,

$$\begin{aligned} B_0 &= 6\% P\left(0, 0.2795\right) + 6\% P\left(0, 1.2795\right) + 106\% P\left(0, 2.2795\right) \\ &= 6\% P\left(0, 0.2795\right) + 6\% \times 0.9716 + 106\% \times 0.9468. \end{aligned} \tag{32}$$

Relativamente ao factor de desconto a 0.2795 anos, via equações (269) e (270) dos apontamentos,

$$\begin{aligned} B\left(0.2795\right) &= \frac{1 - e^{-0.1 \times 0.2795}}{0.1} \\ &\cong 0.2756, \end{aligned}$$

e

$$\begin{aligned} A\left(0.2795\right) &= \left(0.2756 - 0.2795\right)\left(3\% - \frac{0.01^2}{2\left(0.1\right)^2}\right) - \frac{0.01^2}{0.2795 \times 0.1} \times \left(0.2756\right)^2 \\ &\cong -0.0001. \end{aligned}$$

Portanto,

$$\begin{aligned} P\left(0, 0.2795\right) &= \exp\left(-0.0001 - 0.2756 \times 2\%\right) \\ &\cong 0.9944. \end{aligned}$$

Retomando a equação (32), então

$$\begin{aligned} B_0 &= 6\% \times 0.9944 + 6\% \times 0.9716 + 106\% \times 0.9468 \\ &\cong 112.86\%. \end{aligned}$$

(b) Montante de juros vencidos:

$$\begin{aligned} AI &= 6\% \times \frac{263}{365} \\ &\cong 4.32\%. \end{aligned}$$

Consequentemente,

i. $VT^{bid} = 107.50\% + 4.32\% = 111.82\% < B_0 \Longrightarrow$Não vender; e

ii. $VT^{ask} = 107.70\% + 4.32\% = 112.02\% < B_0 \Longrightarrow$Comprar.

(c) Utilizando a Proposição 59 dos apontamentos,

$$\begin{aligned} & p_0\left[P\left(0,1.2795\right);98.20\%;0.2795\right] \\ =\ & -P\left(0,1.2795\right)\Phi\left(-d_1^V\right) + 98.20\% P\left(0,0.2795\right)\Phi\left(-d_0^V\right) \\ =\ & -0.9740 \times \Phi\left(-d_1^V\right) + 0.9820 \times 0.9944 \times \Phi\left(-d_0^V\right), \end{aligned}$$

onde

$$\begin{aligned} v\left(0,0.2795,1.2795\right) &= \sqrt{\frac{0.01^2}{0.1^2}\left[1-e^{-0.1\times 1}\right]^2 \frac{1-e^{-2\times 0.1\times 0.2795}}{2\times 0.1}} \\ &\cong 0.496\%, \end{aligned}$$

$$\begin{aligned} d_1^V &= \frac{\ln\left(\frac{0.9716}{0.9815\times 0.9943}\right) + \frac{(0.496\%)^2}{2}}{0.496\%} \\ &\cong -0.88229269, \end{aligned}$$

e

$$\begin{aligned} d_0^V &= -0.88229269 - 0.496\% \\ &\cong -0.887253812. \end{aligned}$$

Portanto,

$$\begin{aligned} & p_0\left[P\left(0,1.2795\right);98.20\%;0.2795\right] \\ =\ & -0.9740 \times \Phi\left(0.507603154\right) + 0.9820 \times 0.9944 \times \Phi\left(0.512564276\right) \\ =\ & -0.9740 \times 0.694134171 + 0.9820 \times 0.9944 \times 0.695871939 \\ \cong\ & 0.341\%. \end{aligned} \tag{33}$$

(d) De acordo com a Proposição 61 dos apontamentos, o valor actual de uma *put* sobre a CBB pode ser decomposto numa carteira de 2 *puts* Europeias sobre PBD:

$$\begin{aligned} & p_0\left(B_t; X = 108\%; T = 0.2795\right) \\ = \; & 6\% \times p_0\left[P\left(0, 1.2795\right); X_1; T = 0.2795\right] \\ & +106\% \times p_0\left[P\left(0, 2.2795\right); X_2; T = 0.2795\right]. \end{aligned} \tag{34}$$

Os *strikes* podem ser obtidos via equação (327) dos apontamentos:

$$\begin{aligned} X_1 &= \exp\left[A\left(1.2795 - 0.2795\right) - B\left(1\right) \times 1.76\%\right] \\ &= \exp\left(-0.0014 - 0.9516 \times 1.154\%\right) \\ &\cong 98.20\%, \end{aligned}$$

e

$$\begin{aligned} X_2 &= \exp\left[A\left(2.2795 - 0.2795\right) - B\left(2\right) \times 1.76\%\right] \\ &= \exp\left(-0.0055 - 1.8127 \times 1.154\%\right) \\ &\cong 96.33\%. \end{aligned}$$

Portanto,

$$\begin{aligned} & p_0\left(B_t; X = 108\%; T = 0.2795\right) \\ = \; & 6\% \times p_0\left[P\left(0, 1.2795\right); 98.20\%; T = 0.2795\right] \\ & +106\% \times p_0\left[P\left(0, 2.2795\right); 96.33\%; T = 0.2795\right]. \end{aligned} \tag{35}$$

A primeira *put* já foi avaliada na alínea anterior –vide equação (33):

$$p_0\left[P\left(0, 1.2795\right); 98.20\%; T = 0.2795\right] \cong 0.341\%. \tag{36}$$

Relativamente à segunda *put*, o enunciado fornece o seguinte valor actual:

$$p_0\left[P\left(0, 2.2795\right); 96.33\%; T = 0.2795\right] = 0.637\%. \tag{37}$$

Combinando as equações (35), (36) e (37),

$$\begin{aligned} & p_0\left(B_t; X = 108\%; T = 0.2795\right) \\ = \; & 6\% \times 0.341\% + 106\% \times 0.637\% \\ \cong \; & 0.275\%. \end{aligned}$$

# Referências


Shreve, S., 2004, *Stochastic Calculus for Finance II: Continuous-Time Models* (Springer, New York).