Modelos de Estrutura Temporal de Taxas de Juro
Mestrado em Matemática Financeira 05/06
IBS e FCUL
Exame 1ª Época - Resolução

12/Jan/07 Duração: 3h

1. (a) O payoff terminal da Range Digital é dado por:

$$RD_T(S, X_1, X_2, T; M) = M \mathbb{1}_{\{X_1 \leq S_T \leq X_2\}}.$$

Portanto,

$$\begin{aligned} RD_t(S, X_1, X_2, T; M) &= Me^{-r(T-t)} \mathbb{E}_{\mathbb{Q}} \left(\mathbb{1}_{\{X_1 \leq S_T \leq X_2\}} \middle| \mathcal{F}_t \right) \\ &= Me^{-r(T-t)} \mathbb{Q} \left(X_1 \leq S_T \leq X_2 \middle| \mathcal{F}_t \right) \\ &= Me^{-r(T-t)} \left[\mathbb{Q} \left(S_T \leq X_2 \middle| \mathcal{F}_t \right) - \mathbb{Q} \left(S_T \leq X_1 \middle| \mathcal{F}_t \right) \right]. \end{aligned}$$

Finalmente, utilizando a Proposição 13 dos apontamentos,

$$\begin{aligned} & RD_t(S, X_1, X_2, T; M) \\ = {} & Me^{-r(T-t)} \left[Q_{\chi^2\left(\frac{2}{2-\beta}, 2\kappa X_2^{2-\beta}\right)}(2x) - Q_{\chi^2\left(\frac{2}{2-\beta}, 2\kappa X_1^{2-\beta}\right)}(2x) \right]. \end{aligned}$$

(b) O valor actual do derivado sobre volatilidade é dado por:

$$\begin{aligned} c_t(v_t, k, T) &= Me^{-r(T-t)} \mathbb{E}_{\mathbb{Q}} \left[(v_T - k)^+ \middle| \mathcal{F}_t \right] \\ &= Me^{-r(T-t)} \left[\mathbb{E}_{\mathbb{Q}} \left(v_T \mathbb{1}_{\{v_T \geq k\}} \middle| \mathcal{F}_t \right) - k \mathbb{Q} \left(v_T \geq k \middle| \mathcal{F}_t \right) \right], \end{aligned} \tag{1}$$

onde

$$dv_t = k(\theta - v_t)\,dt + \sigma \sqrt{v_t} dW_2^{\mathbb{Q}}(t). \tag{2}$$

Atendendo às equações (7) e (25) dos apontamentos,

$$\left. \frac{v_T}{L} \right| \mathcal{F}_t \sim \chi^2 \left(\frac{4k\theta}{\sigma^2}, b \right), \tag{3}$$

onde

$$b := \frac{4 v_t k}{\sigma^2 \left[e^{k(T-t)} - 1 \right]}, \tag{4}$$

e

$$L := \frac{\sigma^2 \left[1 - e^{-k(T-t)} \right]}{4k}. \tag{5}$$

Consequentemente,

$$
\begin{aligned}
\mathbb{Q}\left(v_T \geq k \middle| \mathcal{F}_t\right) &= \mathbb{Q}\left(\frac{v_T}{L} \geq \frac{k}{L} \middle| \mathcal{F}_t\right) \\
&= Q_{\chi^2\left(\frac{4k\theta}{\sigma^2}, \frac{4v_t k}{\sigma^2\left[e^{k(T-t)}-1\right]}\right)}\left(\frac{k}{L}\right). \qquad (6)
\end{aligned}
$$

Por outro lado e atendendo à Definição 1 dos apontamentos,

$$
\begin{aligned}
& \mathbb{E}_{\mathbb{Q}}\left(v_T \mathbb{1}_{\{v_T \geq k\}} \middle| \mathcal{F}_t\right) \\
= {} & L\mathbb{E}_{\mathbb{Q}}\left(\frac{v_T}{L} \mathbb{1}_{\left\{\frac{v_T}{L} \geq \frac{k}{L}\right\}} \middle| \mathcal{F}_t\right) \\
= {} & L \int_{\frac{k}{L}}^{\infty} x f_{\chi^2\left(\frac{4k\theta}{\sigma^2}, b\right)}(x)\, dx \\
= {} & L \int_0^{\infty} x f_{\chi^2\left(\frac{4k\theta}{\sigma^2}, b\right)}(x)\, dx - L \int_0^{\frac{k}{L}} x f_{\chi^2\left(\frac{4k\theta}{\sigma^2}, b\right)}(x)\, dx \\
= {} & L\mathbb{E}_{\mathbb{Q}}\left[\chi^2\left(\frac{4k\theta}{\sigma^2}, b\right) \middle| \mathcal{F}_t\right] - L\left[x F_{\chi^2\left(\frac{4k\theta}{\sigma^2}, b\right)}(x)\right]_0^{\frac{k}{L}} + L \int_0^{\frac{k}{L}} F_{\chi^2\left(\frac{4k\theta}{\sigma^2}, b\right)}(x)\, dx \\
= {} & L\left(\frac{4k\theta}{\sigma^2} + b\right) - k F_{\chi^2\left(\frac{4k\theta}{\sigma^2}, b\right)}\left(\frac{k}{L}\right) + L \int_0^{\frac{k}{L}} F_{\chi^2\left(\frac{4k\theta}{\sigma^2}, b\right)}(x)\, dx. \qquad (7)
\end{aligned}
$$

Combinando as equações (1), (6) e (7),

$$
\begin{aligned}
c_t\left(v_t, k, T\right) &= M e^{-r(T-t)}\left[L\left(\frac{4k\theta}{\sigma^2} + b\right) - k F_{\chi^2\left(\frac{4k\theta}{\sigma^2}, b\right)}\left(\frac{k}{L}\right) + L \int_0^{\frac{k}{L}} F_{\chi^2\left(\frac{4k\theta}{\sigma^2}, b\right)}(x)\, dx\right. \\
&\qquad \left. - k Q_{\chi^2\left(\frac{4k\theta}{\sigma^2}, b\right)}\left(\frac{k}{L}\right)\right] \\
&= M e^{-r(T-t)}\left[L\left(\frac{4k\theta}{\sigma^2} + b\right) + L \int_0^{\frac{k}{L}} F_{\chi^2\left(\frac{4k\theta}{\sigma^2}, b\right)}(x)\, dx - k\right].
\end{aligned}
$$

2. (a) O valor actual do *caplet* é dado por:

$$
\begin{aligned}
Caplet_0 &= EUR1M \times (1 + 4.5\%) \times p_0\left[P(0,2); \frac{1}{1 + 4.5\%}; 1\right] \\
&= EUR1M \times 1.045 \times p_0\left[P(0,2); 95.694\%; 1\right]. \qquad (8)
\end{aligned}
$$

Utilizando a Proposição 59 dos apontamentos,

$$
\begin{aligned}
& p_0\left[P(0,2); 95.694\%; 1\right] \\
= {} & -P(0,2)\, \Phi\left(-d_1^V\right) + 95.694\% \times P(0,1)\, \Phi\left(-d_0^V\right) \\
= {} & -0.9150 \times \Phi\left(-d_1^V\right) + 95.694\% \times 0.9608 \times \Phi\left(-d_0^V\right),
\end{aligned}
$$

onde

$$\begin{aligned} v(0,1,2) &= \sqrt{\frac{0.05^2}{0.5^2}\left[1-e^{-0.5(2-1)}\right]^2 \frac{1-e^{-2\times 0.5\times 1}}{2\times 0.5}} \\ &\cong 6.257\%, \end{aligned}$$

$$\begin{aligned} d_1^V &= \frac{\ln\left[\frac{0.9150}{95.694\%\times 0.9608}\right]+\frac{(6.257\%)^2}{2}}{6.257\%} \\ &\cong -0.045743962, \end{aligned}$$

e

$$\begin{aligned} d_0^V &= -0.045743962 - 6.257\% \\ &\cong -0.108310317. \end{aligned}$$

Portanto,

$$\begin{aligned} & p_0\left[P(0,2);95.694\%;1\right] \\ = \;& -0.9150\times\Phi(0.045743962)+95.694\%\times 0.9608\times\Phi(0.108310317) \\ = \;& -0.9150\times 0.518242838+95.694\%\times 0.9608\times 0.54312523 \\ \cong \;& 2.517\%. \end{aligned} \tag{9}$$

Combinando as equações (8) e (9),

$$\begin{aligned} Caplet_0 &= EUR1M\times 1.045\times 2.517\% \\ &\cong EUR26,301.54. \end{aligned}$$

(b) De acordo com a Proposição 61 dos apontamentos, o valor actual da *put* sobre a CBB pode ser decomposto numa carteira de 2 *puts* Europeias sobre PBD:

$$\begin{aligned} & p_0\left(B_t;X=98.24\%;T=1\right) \\ = \;& 4\%\times p_0\left[P(0,2);X_1;T=1\right]+104\%\times p_0\left[P(0,3);X_2;T=1\right]. \end{aligned}$$

Os *strikes* podem ser obtidos via equação (327) dos apontamentos:

$$\begin{aligned} X_1 &= \exp\left[A(2-1)-B(2-1)\times 3.915\%\right] \\ &= \exp\left(-0.0058-0.9754\times 3.915\%\right) \\ &\cong 95.694\%, \end{aligned}$$

e

$$\begin{aligned} X_2 &= \exp\left[A(3-1)-B(3-1)\times 3.915\%\right] \\ &= \exp\left(-0.0222-1.9033\times 3.915\%\right) \\ &\cong 90.783\%. \end{aligned}$$

Portanto,

$$\begin{aligned} & p_0\left(B_t; X = 98.24\%; T = 1\right) \\ = \ & 4\% \times p_0\left[P\left(0,2\right); 95.694\%; T = 1\right] + 104\% \times p_0\left[P\left(0,3\right); 90.783\%; T = 1\right] \end{aligned} \tag{10}$$

A primeira *put* já foi avaliada na alínea anterior –vide equação (9)

$$p_0\left[P\left(0,2\right); 95.694\%; T = 1\right] \cong 2.517\%. \tag{11}$$

Relativamente à segunda *put*:

$$\begin{aligned} & p_0\left[P\left(0,3\right); 90.783\%; T = 1\right] \\ = \ & -P\left(0,3\right)\Phi\left(-d_1^V\right) + 90.783\% \times P\left(0,1\right)\Phi\left(-d_0^V\right) \\ = \ & -0.8650 \times \Phi\left(-d_1^V\right) + 90.783\% \times 0.9608 \times \Phi\left(-d_0^V\right), \end{aligned}$$

onde

$$\begin{aligned} v\left(0,1,3\right) &= \sqrt{\frac{0.05^2}{0.5^2}\left[1 - e^{-0.5(3-1)}\right]^2 \frac{1 - e^{-2\times0.5\times1}}{2\times0.5}} \\ &\cong 10.051\%, \end{aligned}$$

$$\begin{aligned} d_1^V &= \frac{\ln\left[\frac{0.8650}{90.783\%\times0.9608}\right] + \frac{(10.051\%)^2}{2}}{10.051\%} \\ &\cong -0.033056494, \end{aligned}$$

e

$$\begin{aligned} d_0^V &= -0.033056494 - 10.051\% \\ &\cong -0.13357126. \end{aligned}$$

Portanto,

$$\begin{aligned} & p_0\left[P\left(0,3\right); 90.783\%; T = 1\right] \\ = \ & -0.8650 \times \Phi\left(0.033056494\right) + 90.783\% \times 0.9608 \times \Phi\left(0.13357126\right) \\ = \ & -0.8650 \times 0.513185232 + 90.783\% \times 0.9608 \times 0.553129194 \\ \cong \ & 3.857\%. \end{aligned} \tag{12}$$

Combinando as equações (10), (11) e (12):

$$\begin{aligned} p_0\left(B_t; X = 98.24\%; T = 1\right) &= 4\% \times 2.517\% + 104\% \times 3.857\% \\ &\cong 4.112\%. \end{aligned}$$

(c) Assumindo o processo estocástico

$$dr_t = \alpha\left(\gamma - r_t\right)dt + \rho dW_t^{\mathbb{Q}},$$

e utilizando a Proposição 37 dos apontamentos,

$$r_s = e^{-\alpha(s-t)} r_t + \gamma \left[1 - e^{-\alpha(s-t)}\right] + \rho \int_t^s e^{-\alpha(s-u)} dW_u^{\mathbb{Q}}. \tag{13}$$

Portanto,

$$\int_t^T r_s ds = r_t \int_t^T e^{-\alpha(s-t)} ds + \gamma \int_t^T \left[1 - e^{-\alpha(s-t)}\right] ds + \rho \int_t^T \left[\int_t^s e^{-\alpha(s-u)} dW_u^{\mathbb{Q}}\right] ds. \tag{14}$$

Uma vez que

$$\begin{aligned} & \{(s,u) : t \leq s \leq T, t \leq u \leq s\} \\ = & \{(s,u) : t \leq u \leq T, u \leq s \leq T\}, \end{aligned}$$

então:

$$\int_t^T \left[\int_t^s e^{-\alpha(s-u)} dW_u^{\mathbb{Q}}\right] ds = \int_t^T \left[\int_u^T e^{-\alpha(s-u)} ds\right] dW_u^{\mathbb{Q}}. \tag{15}$$

Combinando as equações (14) e (15),

$$\int_t^T r_s ds = r_t \int_t^T e^{-\alpha(s-t)} ds + \gamma \int_t^T \left[1 - e^{-\alpha(s-t)}\right] ds + \rho \int_t^T \left[\int_u^T e^{-\alpha(s-u)} ds\right] dW_u^{\mathbb{Q}}. \tag{16}$$

Consequentemente,

$$\mathbb{E}_{\mathbb{Q}}\left(\int_t^T r_s ds \middle| \mathcal{F}_t\right) = r_t \int_t^T e^{-\alpha(s-t)} ds + \gamma \int_t^T \left[1 - e^{-\alpha(s-t)}\right] ds,$$

e portanto,

$$\begin{aligned} & \mathbb{E}_{\mathbb{Q}}\left\{\left[\int_t^T r_s ds - \mathbb{E}_{\mathbb{Q}}\left(\int_t^T r_s ds \middle| \mathcal{F}_t\right)\right]^2 \middle| \mathcal{F}_t\right\} \\ = & \mathbb{E}_{\mathbb{Q}}\left\{\left[\rho \int_t^T \left(\int_u^T e^{-\alpha(s-u)} ds\right) dW_u^{\mathbb{Q}}\right]^2 \middle| \mathcal{F}_t\right\} \\ = & \rho^2 \int_t^T \left(\int_u^T e^{-\alpha(s-u)} ds\right)^2 du. \end{aligned}$$

3. (a) Via equação (343) dos apontamentos,

$$\begin{aligned} \gamma &= \sqrt{2^2 + 2 \times (10\%)^2} \\ &\cong 2.004993766, \end{aligned}$$

e

$$\begin{aligned} B(3) &= -\frac{2\left(e^{2.004993766 \times 3} - 1\right)}{2 \times 2.004993766 + (2 + 2.004993766)\left(e^{2.004993766 \times 3} - 1\right)} \\ &\cong -0.4982. \end{aligned}$$

Portanto,

$$\begin{aligned} P(0,3) &= \exp\left[-0.1000 - 0.4982 \times 5\%\right] \\ &\cong 0.8826. \end{aligned}$$

Consequentemente, a taxa *spot* a 3 anos com capitalização contínua é igual a:

$$\begin{aligned} r_c(0,3) &= -\frac{\ln(0.8826)}{3} \\ &\cong 4.162\%. \end{aligned}$$

(b) O *payoff* final da *asset-or-nothing European call* em apreço é dado por:

$$c_{T_1}^{AN}\left[P(T_1,T_2);K;T_1\right] = P(T_1,T_2)\,\mathbb{1}_{\{P(T_1,T_2)>K\}}, \tag{17}$$

onde $T_2 = 2$ anos, $K = 94.018\%$, and $T_1 = 0.5$ anos.
Consequentemente,

$$\begin{aligned} c_t^{AN}\left[P(t,T_2);K;T_1\right] &= P(t,T_2)\,\mathbb{E}_{\mathbb{Q}_2}\left[\frac{P(T_1,T_2)\,\mathbb{1}_{\{P(T_1,T_2)>K\}}}{P(T_1,T_2)}|\mathcal{F}_t\right] \\ &= P(t,T_2)\,\mathbb{Q}_2\left[P(T_1,T_2) > K|\mathcal{F}_t\right] \\ &= P(t,T_2)\,\mathbb{Q}_2\left[A(T_2-T_1) + B(T_2-T_1)\,r_{T_1} > \ln(K)\,|\mathcal{F}_t\right] \\ &= P(t,T_2)\,\mathbb{Q}_2\left(r_{T_1} < r^*|\mathcal{F}_t\right) \\ &= P(t,T_2)\,\mathbb{Q}_2\left(\frac{r_{T_1}}{L_2} < \frac{r^*}{L_2}|\mathcal{F}_t\right) \\ &= P(t,T_2)\,F_{\chi^2_{\left(\frac{4k\theta}{\sigma^2},\zeta_2\right)}}\left(\frac{r^*}{L_2}\right). \end{aligned} \tag{18}$$

Atendendo aos dados do enunciado, $P(t,T_2) = P(0,2) = 0.9187$,

$$\begin{aligned} r^* &= \frac{\ln(K) - A(T_2-T_1)}{B(T_2-T_1)} \\ &= \frac{\ln(94.018\%) - A(2-0.5)}{B(1.5)} \\ &= \frac{\ln(94.018\%) + 0.0410}{-0.4747} \\ &\cong 4.363\%, \end{aligned}$$

$$\begin{aligned} L_2 &= \frac{\sigma^2}{2}\frac{e^{\gamma(T_1-t)} - 1}{\gamma\left[e^{\gamma(T_1-t)}+1\right] + \left[k - \sigma^2 B(T_2-T_1)\right]\left[e^{\gamma(T_1-t)}-1\right]} \\ &= \frac{\frac{0.1^2}{2} \times \left(e^{2.004993766\times 0.5} - 1\right)}{2.004993766 \times \left[e^{2.004993766\times 0.5}+1\right] + \left[2 - 0.1^2 \times (-0.4747)\right]\left[e^{2.004993766\times 0.5}-1\right]} \\ &\cong 0.000789357, \end{aligned}$$

e

$$
\begin{aligned}
\zeta_2 &= \frac{8 r_t \gamma^2 e^{\gamma(T_1-t)}}{\sigma^2\left[e^{\gamma(T_1-t)}-1\right]\left\{\gamma\left[e^{\gamma(T_1-t)}+1\right]+\left[k-\sigma^2 B\left(T_2-T_1\right)\right]\left[e^{\gamma(T_1-t)}-1\right]\right\}} \\
&= \left[8\times 5\%\times(2.004993766)^2\times e^{2.004993766\times 0.5}\right]\Big\{0.1^2\times\left(e^{2.004993766\times 0.5}-1\right) \\
&\quad \Big[2.004993766\times\left(e^{2.004993766\times 0.5}+1\right)+\left(2-0.1^2\times(-0.4747)\right)\left(e^{2.004993766\times 0.5}-1\right)\Big]\Big\} \\
&\cong 23.2461663.
\end{aligned}
$$

Utilizando a equação (18),

$$
\begin{aligned}
& c_0^{AN}\left[P(0,2); K=94.018\%; T_1=0.5\right] \\
=\ & 0.9187\times F_{\chi^2_{\left(\frac{4\times 2\times 4\%}{0.1^2},23.2461663\right)}}\left(\frac{4.363\%}{0.000789357}\right) \\
=\ & 0.9187\times F_{\chi^2_{(32,23.2461663)}}(55.27559522).
\end{aligned}
$$

A probabilidade contida na equação anterior pode ser calculada via aproximação de Sankaran, i.e.

$$
\begin{aligned}
F_{\chi^2(a,b)}(z) &= \mathbb{P}\left(\chi^2(a,b)\le z\right) \\
&= \mathbb{P}\left\{\left[\frac{\chi^2(a,b)}{a+b}\right]^h \le \left(\frac{z}{a+b}\right)^h\right\} \\
&\approx \Phi\left[\frac{\left(\frac{z}{a+b}\right)^h-\mu_h}{\sigma_h}\right],
\end{aligned}
\tag{19}
$$

onde

$$
\mu_h := 1+h(h-1)\frac{a+2b}{(a+b)^2}-h(h-1)(2-h)(1-3h)\frac{(a+2b)^2}{2(a+b)^4}, \tag{20}
$$

$$
\sigma_h^2 := h^2\frac{2(a+2b)}{(a+b)^2}\left[1-(1-h)(1-3h)\frac{a+2b}{(a+b)^2}\right], \tag{21}
$$

e

$$
h := 1-\frac{2}{3}(a+b)(a+3b)(a+2b)^{-2}. \tag{22}
$$

Uma vez que $a=32$, $b=23.2461663$, então

$$
\begin{aligned}
h &= 1-\frac{2}{3}(32+23.2461663)(32+3\times 23.2461663)(32+2\times 23.2461663)^{-2} \\
&\cong 0.391806545,
\end{aligned}
$$

$\mu_h = 0.993849521$, and $\sigma_h^2 = 0.007917467$. Portanto,

$$
\begin{aligned}
& c_0^{AN}\left[P(0,2); K=94.018\%; T_1=0.5\right] \\
\approx\ & 0.9187\times\Phi\left[\frac{\left(\frac{55.27559522}{32+23.2461663}\right)^{0.391806545}-0.993849521}{\sqrt{0.007917467}}\right] \\
=\ & 0.9187\times\Phi(0.071467123) \\
=\ & 0.9187\times 0.528487005 \\
\cong\ & 0.485499585.
\end{aligned}
\tag{23}
$$

4. (a) Visto que $\Delta t = 0.25$, então

$$\begin{aligned}\Delta r &= 2\%\sqrt{3 \times 0.25} \\ &\cong 1.732\%.\end{aligned}$$

Portanto,

$$\begin{aligned}r^*_{4,1} &= 1 \times 1.732\% \\ &\cong 1.732\%.\end{aligned}$$

Em alternativa $r^*_{4,1} = r^*_{3,1} = r^*_{2,1} = r^*_{1,1}$.
Dado que

$$\begin{aligned}\frac{0.184}{a\Delta t} &= \frac{0.184}{0.4 \times 0.25} \\ &\cong 1.84,\end{aligned}$$

então $j_{\max} = 2$. Consequentemente, temos de considerar um *normal branching process* a partir do nó $(4, 1)$:

$$\begin{aligned}p_u(4,1) &= \frac{1}{6} + \frac{a^2 j^2 (\Delta t)^2 - aj\Delta t}{2} \\ &= \frac{1}{6} + \frac{0.4^2 1^2 (0.25)^2 - 0.4 \times 1 \times 0.25}{2} \\ &\cong 0.103,\end{aligned}$$

$$\begin{aligned}p_m(4,1) &= \frac{2}{3} - a^2 j^2 (\Delta t)^2 \\ &= \frac{2}{3} - 0.4^2 1^2 (0.25)^2 \\ &\cong 0.657,\end{aligned}$$

e

$$\begin{aligned}p_d(4,1) &= 1 - p_u(4,1) - p_m(4,1) \\ &= 1 - 0.103 - 0.657 \\ &\cong 0.222.\end{aligned}$$

(b) Começando pelo *Arrow-Debreu price*:

$$\begin{aligned}Q(4,-1) &= Q(3,0)\, p_d(3,0) \exp(-r_{3,0}\Delta t) + Q(3,-1)\, p_m(3,-1) \exp(-r_{3,-1}\Delta t) \\ &\quad + Q(3,-2)\, p_m(3,-2) \exp(-r_{3,-2}\Delta t) \\ &= 0.4340 \times 0.167 \times \exp(-4.363\% \times 0.25) \\ &\quad + 0.2260 \times 0.657 \times \exp(-2.631\% \times 0.25) \\ &\quad + 0.0439 \times 0.027 \times \exp(-0.899\% \times 0.25) \\ &\cong 0.2202.\end{aligned}$$

Relativamente à taxa "instantânea", comecemos por determinar $\alpha(4)$ utilizando, por exemplo, $r_{4,0}$:

$$\begin{aligned} r_{4,0} &= \alpha(4) + 0 \times \Delta r \\ &\Longrightarrow \alpha(4) = 4.59\%. \end{aligned}$$

Portanto,

$$\begin{aligned} r_{4,-1} &= \alpha(4) - 1 \times \Delta r \\ &= 4.59\% - 1.732\% \\ &= 2.858\%. \end{aligned}$$

(c) Visto o modelo efectuar o fit automático à curva de taxas de juro vigente em mercado,

$$\begin{aligned} AC_{0,0} &= \max\left[P(0,1y) - 97.903\%, \exp(-r_{0,0}\Delta t)\left(p_u AC_{1,1} + p_m AC_{1,0} + p_d AC_{1,-1}\right)\right] \\ &= \max[96.08\% - 97.903\%, \\ &\quad \exp(-3.629\% \times 0.25)(0.167 \times 0 + 0.667 \times 0.0013 + 0.167 \times 0.0072)] \\ &\cong 0.00207179. \end{aligned}$$