Modelos de Estrutura Temporal de Taxas de Juro
Mestrado em Matemática Financeira 10/11
IBS e FCUL
Exame 1ª Época - Resolução

20/Dez/11 Duração: 3h

1. (a) Integrando ambos os membros da equação diferencial ordinária

$$\frac{\partial \phi_{\lambda,\mu}(t-t_0)}{\partial t} = -k\theta\psi_{\lambda,\mu}(t-t_0),$$

obtém-se

$$\phi_{\lambda,\mu}(t-t_0) - \phi_{\lambda,\mu}(0) = -k\theta\int_{t_0}^{t}\psi_{\lambda,\mu}(u-t_0)\,du,$$

i.e.

$$\phi_{\lambda,\mu}(t-t_0) = -k\theta\int_{t_0}^{t}\frac{\lambda\left[h+k+(h-k)e^{h(u-t_0)}\right]+2\mu\left[e^{h(u-t_0)}-1\right]}{g(u)}du, \quad (1)$$

sendo

$$g(u) := \sigma^2\lambda\left(e^{h(u-t_0)}-1\right)+h-k+(k+h)e^{h(u-t_0)}. \quad (2)$$

Visto que

$$h=\sqrt{k^2+2\sigma^2\mu},$$

então o numerador da função integranda na equação (1) pode ser reescrito como

$$\begin{aligned}
& \lambda\left[h+k+(h-k)e^{h(u-t_0)}\right]+2\mu\left[e^{h(u-t_0)}-1\right] \\
= & \left[2\mu+\lambda(h-k)\right]e^{h(u-t_0)}+\left[\lambda(h+k)-2\mu\right] \\
= & \left[\frac{h^2-k^2}{\sigma^2}+\lambda(h-k)\right]e^{h(u-t_0)}+\left[\lambda(h+k)-\frac{h^2-k^2}{\sigma^2}\right] \\
= & -\frac{h+k}{\sigma^2}\left\{\left[-(h-k)-\lambda\frac{(h-k)\sigma^2}{h+k}\right]e^{h(u-t_0)}+\left[-\lambda\sigma^2+(h-k)\right]\right\} \\
= & -\frac{h+k}{\sigma^2}\left\{\left[-(h-k)-\lambda\frac{(h-k)\sigma^2}{h+k}-\sigma^2\lambda-(k+h)\right]e^{h(u-t_0)}+g(u)\right\} \\
= & -\frac{h+k}{\sigma^2}\left\{\left[-2h-\lambda\frac{(h-k)\sigma^2}{h+k}-\sigma^2\lambda\right]e^{h(u-t_0)}+g(u)\right\}, \quad (3)
\end{aligned}$$

sendo que a penúltima igualdade resulta da definição (2).
Combinando as equações (1) e (3), e atendendo a que

$$\frac{\partial g(u)}{\partial u} = \left(\sigma^2\lambda+k+h\right)he^{h(u-t_0)},$$

obtém-se

$$
\begin{aligned}
& \phi_{\lambda,\mu}(t-t_0) \\
= & -k\theta\frac{h+k}{\sigma^2}\int_{t_0}^{t}\frac{\left[2h+\lambda\frac{(h-k)\sigma^2}{h+k}+\sigma^2\lambda\right]e^{h(u-t_0)}-g(u)}{g(u)}du \\
= & k\theta\frac{h+k}{\sigma^2}(t-t_0)-k\theta\frac{h+k}{\sigma^2}\frac{2h+\lambda\frac{(h-k)\sigma^2}{h+k}+\sigma^2\lambda}{(\sigma^2\lambda+k+h)h}\int_{t_0}^{t}\frac{\frac{\partial g(u)}{\partial u}}{g(u)}du \\
= & \frac{k\theta}{\sigma^2}(h+k)(t-t_0)-\frac{k\theta}{\sigma^2}\frac{2h(h+k)+\lambda(h-k)\sigma^2+\sigma^2\lambda(h+k)}{(\sigma^2\lambda+k+h)h}\ln g(u)\big|_{t_0}^{t} \\
= & \frac{k\theta}{\sigma^2}(h+k)(t-t_0)-\frac{k\theta}{\sigma^2}\frac{2h(h+k+\sigma^2\lambda)}{(\sigma^2\lambda+k+h)h}\ln\left[\frac{g(t)}{h-k+(k+h)}\right] \\
= & \frac{2k\theta}{\sigma^2}\frac{(h+k)}{2}(t-t_0)-\frac{2k\theta}{\sigma^2}\ln\left[\frac{g(t)}{2h}\right] \\
= & \frac{2k\theta}{\sigma^2}\ln\left[\frac{2he^{\frac{(h+k)}{2}(t-t_0)}}{g(t)}\right].
\end{aligned}
$$

(b) Como o futuro é um $\mathbb{Q}$-martingale, então

$$F_t = M\times\delta\times\mathbb{E}_{\mathbb{Q}}\left[100\% - E(T,T+\delta)\right|\mathcal{F}_t].$$

Por outro lado, como

$$P(T,T+\delta)=\frac{1}{1+\delta\times E(T,T+\delta)},$$

então

$$
\begin{aligned}
F_t &= M\times\delta\times\mathbb{E}_{\mathbb{Q}}\left\{1-\frac{1}{\delta}\left[\frac{1}{P(T,T+\delta)}-1\right]\middle|\mathcal{F}_t\right\} \\
&= M\times\left\{1+\delta-\mathbb{E}_{\mathbb{Q}}\left[P(T,T,T+\delta)^{-1}\middle|\mathcal{F}_t\right]\right\}. \qquad (4)
\end{aligned}
$$

Utilizando a equação (315) dos apontamentos, sabemos que

$$
\begin{aligned}
P(T,T,T+\delta)^{-1} = & \frac{P(t,T)}{P(t,T+\delta)}\exp\left\{\frac{\rho^2}{2}\int_t^T\left[B^2(T+\delta-s)-B^2(T-s)\right]ds\right. \\
& \left.+\rho\int_t^T\left[B(T+\delta-s)-B(T-s)\right]dW_s^{\mathbb{Q}}\right\}.
\end{aligned}
$$

Visto que

$$\rho\int_t^T\left[B(T+\delta-s)-B(T-s)\right]dW_s^{\mathbb{Q}}\bigg|\mathcal{F}_t \overset{\mathbb{Q}}{\sim} N^1\left(0,v^2(t,T,T+\delta)\right),$$

então

$$
\begin{aligned}
& \mathbb{E}_{\mathbb{Q}}\left[P\left(T,T,T+\delta\right)^{-1} \middle| \mathcal{F}_t\right] && (5)\\
= \; & \frac{P\left(t,T\right)}{P\left(t,T+\delta\right)} \exp\left\{ \frac{\rho^2}{2} \int_t^T \left[B^2\left(T+\delta-s\right) - B^2\left(T-s\right)\right] ds + 0 + \frac{v^2\left(t,T,T+\delta\right)}{2} \right\}.
\end{aligned}
$$

Finalmente, combinando as equações (4) e (5), obtemos:

$$
\begin{aligned}
F_t \; = \; & M \times (1+\delta) - M \times P\left(t,T,T+\delta\right)^{-1} \\
& \exp\left\{ \frac{\rho^2}{2} \int_t^T \left[B^2\left(T+\delta-s\right) - B^2\left(T-s\right)\right] ds + \frac{v^2\left(t,T,T+\delta\right)}{2} \right\}.
\end{aligned}
$$

2. Via equação (59) dos handouts, o valor da put Europeia (com $\beta < 2$) é dado por:

$$
p_t\left(S,X,T\right) = -S_t e^{-q(T-t)} F_{\chi^2\left(2+\frac{2}{2-\beta},2x\right)}\left(2\kappa X^{2-\beta}\right) + X e^{-r(T-t)} Q_{\chi^2\left(\frac{2}{2-\beta},2\kappa X^{2-\beta}\right)}\left(2x\right), \tag{6}
$$

$$
\kappa := \frac{2\left(r-q\right)}{\left(2-\beta\right)\delta^2\left[e^{(2-\beta)(r-q)(T-t)}-1\right]}, \tag{7}
$$

e

$$
x := \kappa S_t^{2-\beta} e^{(2-\beta)(r-q)(T-t)}. \tag{8}
$$

Sendo a volatilidade da taxa de rentabilidade do activo subjacente é igual a 30% ao ano, então, e via equação (2) dos handouts,

$$
\delta = \frac{30\%}{(10)^{\frac{-3-2}{2}}} = 94.868330.
$$

Retomando as equações (7) e (8),

$$
\kappa = \frac{2\left(2\% - 1\%\right)}{\left(2+3\right)\left(94.86833\right)^2\left[e^{(2+3)(2\%-1\%)\times 0.5}-1\right]} \cong 1.75565 \times 10^{-5},
$$

e

$$
x = 1.75565 \times 10^{-5} \times (10)^{2+3} e^{(2+3)(2\%-1\%)\times 0.5} \cong 1.800092592.
$$

Substituindo na equação (6), então

$$
\begin{aligned}
p_t \; = \; & -10 \times e^{-1\%\times 0.5} \times F_{\chi^2\left(2+\frac{2}{2+3},2\times 1.800092592\right)}\left(2 \times 1.75565 \times 10^{-5} \times 8^{2+3}\right) \\
& +8 \times e^{-2\%\times 0.5} \times Q_{\chi^2\left(\frac{2}{2+3},2\times 1.75565\times 10^{-5}\times 8^{2+3}\right)}\left(2 \times 1.800092592\right) \\
= \; & -10 \times e^{-1\%\times 0.5} \times F_{\chi^2(2.4,3.600185183)}\left(1.15058157\right) && (9)\\
& +8 \times e^{-2\%\times 0.5} \times Q_{\chi^2(0.4,1.15058157)}\left(3.600185183\right).
\end{aligned}
$$

Via tabela do enunciado, sabemos que

$$
F_{\chi^2(2.4,3.600185183)}\left(1.15058157\right) = 0.914942118. \tag{10}
$$

(a) A probabilidade $Q_{\chi^2(0.4,1.15058157)}(3.600185183)$ pode ser calculada via aproximação de Sankaran, i.e.

$$\begin{aligned} Q_{\chi^2(a,b)}(z) &= 1 - \mathbb{Q}\left(\chi^2(a,b) < z\right) \\ &= 1 - \mathbb{Q}\left\{\left[\frac{\chi^2(a,b)}{a+b}\right]^h < \left(\frac{z}{a+b}\right)^h\right\} \\ &\approx \Phi\left[-\frac{\left(\frac{z}{a+b}\right)^h - \mu_h}{\sigma_h}\right], \end{aligned} \tag{11}$$

onde

$$\mu_h := 1 + h(h-1)\frac{a+2b}{(a+b)^2} - h(h-1)(2-h)(1-3h)\frac{(a+2b)^2}{2(a+b)^4}, \tag{12}$$

$$\sigma_h^2 := h^2 \frac{2(a+2b)}{(a+b)^2}\left[1 - (1-h)(1-3h)\frac{a+2b}{(a+b)^2}\right], \tag{13}$$

e

$$h := 1 - \frac{2}{3}(a+b)(a+3b)(a+2b)^{-2}. \tag{14}$$

Visto que $a = 0.4$ e $b = 1.15058157$, então

$$\begin{aligned} h &= 1 - \frac{2}{3}(0.4 + 1.15058157)(0.4 + 3 \times 1.15058157) \\ &\quad (0.4 + 2 \times 1.15058157)^{-2} \\ &\cong 0.454293378, \end{aligned}$$

$$\begin{aligned} \mu_h &= 1 + 0.4542933784 \times (0.454293378 - 1)\frac{0.4 + 2 \times 1.15058157}{(0.4 + 1.15058157)^2} \\ &\quad -0.454293378 \times (0.454293378 - 1)(2 - 0.4542933784) \\ &\quad (1 - 3 \times 0.454293378)\frac{(0.4 + 2 \times 1.15058157)^2}{2(0.4 + 1.15058157)^4} \\ &\cong 0.633723021, \end{aligned}$$

e

$$\begin{aligned} \sigma_h^2 &= 0.454293378^2 \times \frac{2(0.4 + 2 \times 1.15058157)}{(0.4 + 1.15058157)^2} \\ &\quad \left[1 - (1 - 0.454293378)(1 - 3 \times 0.454293378)\frac{0.4 + 2 \times 1.15058157}{(0.4 + 1.15058157)^2}\right] \\ &\cong 0.566897941. \end{aligned}$$

Utilizando a equação (11),

$$\Phi\left[-\frac{\left(\frac{z}{a+b}\right)^h - \mu_h}{\sigma_h}\right]$$

$$
\begin{aligned}
Q_{\chi^2(0.4,1.15058157)}(3.600185183) &= \Phi\left[-\frac{\left(\frac{3.6001851833}{0.4+1.15058157}\right)^{0.454293378}-0.633723021}{\sqrt{0.566897941}}\right] \\
&\cong 0.134436848. \qquad (15)
\end{aligned}
$$

Finalmente, combinando as equações (9), (10) e (15),

$$
\begin{aligned}
p_t &= -10\times e^{-1\%\times 0.5}\times 0.914942118+8\times e^{-2\%\times 0.5}\times 0.134436848 \\
&\cong EUR0.21846.
\end{aligned}
$$

3. Utilizando a Proposição 22 dos apontamentos,

$$
\begin{aligned}
p_0 &= -10\times e^{-1\%\times 0.5}\times\left[1-P_1\left(S_t=10,v_t=0.09;T=0.5,X=8\right)\right] \qquad (16)\\
&\quad +e^{-2\%\times 0.5}\times 8\times\left[1-P_2\left(S_t=10,v_t=0.09;T=0.5,X=8\right)\right].
\end{aligned}
$$

Com base nas equações (173) e (174) dos apontamentos:

$$
\begin{aligned}
P_1\left(S_t=10,v_t=0.09;T=0.5,X=8\right) &\approx \frac{1}{2}+\frac{1.22557595}{\pi} \\
&\cong 0.89011, \qquad (17)
\end{aligned}
$$

e

$$
\begin{aligned}
P_2\left(S_t=10,v_t=0.09;T=0.5,X=8\right) &\approx \frac{1}{2}+\frac{1.08032970}{\pi} \\
&\cong 0.84388. \qquad (18)
\end{aligned}
$$

(a) Combinando as equações (16), (17) e (18),

$$
\begin{aligned}
p_0 &= -10\times e^{-1\%\times 0.5}\times(1-0.89011)+e^{-2\%\times 0.5}\times 8\times(1-0.84388) \\
&\cong EUR0.14315.
\end{aligned}
$$

4. (a) Utilizando a Proposição 59 dos apontamentos,

$$
\begin{aligned}
& c_0\left[P(0,1.5);98.095\%;0.5\right] \\
&= P(0,1.5)\,\Phi\left(d_1^V\right)-98.095\%P(0,0.5)\,\Phi\left(d_0^V\right) \\
&= 0.9753\times\Phi\left(d_1^V\right)-0.98095\times 0.9932\times\Phi\left(d_0^V\right),
\end{aligned}
$$

onde

$$
\begin{aligned}
v(0,0.5,1.5) &= \sqrt{\frac{0.05^2}{2^2}\left[1-e^{-2\times(1.5-0.5)}\right]^2\frac{1-e^{-2\times 2\times 0.5}}{2\times 2}} \\
&\cong 1.005\%,
\end{aligned}
$$

$$
\begin{aligned}
d_1^V &= \frac{\ln\left(\frac{0.9753}{0.98095\times 0.9932}\right)+\frac{(1.005\%)^2}{2}}{1.005\%} \\
&\cong 0.109272841,
\end{aligned}
$$

e

$$\begin{aligned} d_0^V &= 0.109272841 - 1.005\% \\ &\cong 0.099222481. \end{aligned}$$

Portanto,

$$\begin{aligned} & c_0\left[P(0,1.5);98.095\%;0.5\right] \\ = \ & 0.9753 \times \Phi(0.109272841) - 0.98095 \times 0.9932 \times \Phi(0.099222481) \\ = \ & 0.9753 \times 0.543506957 - 0.98095 \times 0.9932 \times 0.539519187 \\ \cong \ & 0.444\%. \end{aligned} \tag{19}$$

(b) De acordo com a Proposição 61 dos apontamentos, o valor actual de uma *call* sobre a CBB pode ser decomposto numa carteira de 2 *calls* Europeias sobre PBD:

$$\begin{aligned} & c_0\left(B_t; X = 104\%; T = 0.5\right) \\ = \ & \frac{4\%}{2} \times c_0\left[P(0,1); X_1; T = 0.5\right] \\ & + \left(\frac{4\%}{2} + 102\%\right) \times c_0\left[P(0,1.5); X_2; T = 0.5\right]. \end{aligned} \tag{20}$$

Os *strikes* podem ser obtidos via equação (327) dos apontamentos:

$$\begin{aligned} X_1 &= \exp\left[A(1-0.5) - B(0.5) \times 1.85\%\right] \\ &= \exp\left(-0.0037 - 0.3161 \times 1.85\%\right) \\ &\cong 99.055\%, \end{aligned}$$

e

$$\begin{aligned} X_2 &= \exp\left[A(1.5-0.5) - B(1) \times 1.85\%\right] \\ &= \exp\left(-0.011234 - 0.4323 \times 1.85\%\right) \\ &\cong 98.095\%. \end{aligned}$$

Portanto,

$$\begin{aligned} & c_0\left(B_t; X = 104\%; T = 0.5\right) \\ = \ & \frac{4\%}{2} \times c_0\left[P(0,1); 99.055\%; T = 0.5\right] \\ & + \left(\frac{4\%}{2} + 102\%\right) \times c_0\left[P(0,1.5); 98.095\%; T = 0.5\right]. \end{aligned} \tag{21}$$

A segunda *call* já foi avaliada na alínea anterior –vide equação (19):

$$c_0\left[P(0,1.5);98.095\%;0.5\right] \cong 0.444\%. \tag{22}$$

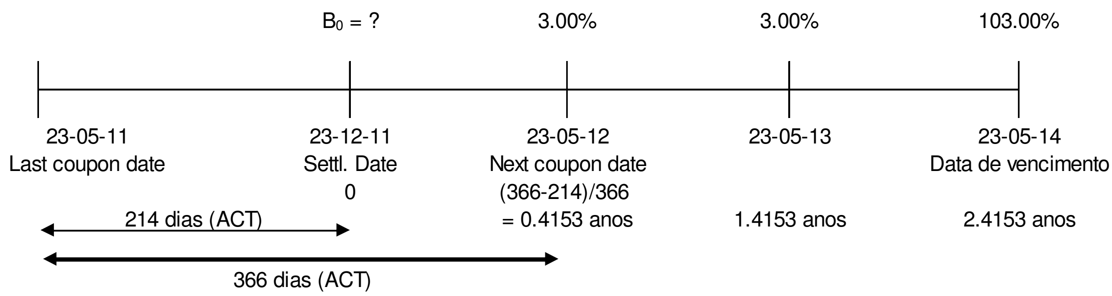
Relativamente à primeira *call*, o enunciado fornece o seguinte valor actual:

$$c_0\left[P(0,1);99.055\%;T=0.5\right]=0.327\%. \tag{23}$$

Combinando as equações (21), (22) e (23),

$$\begin{aligned}&c_0\left(B_t;X=104\%;T=0.5\right)\\ =\;& 2\%\times 0.327\%+104\%\times 0.444\%\\ \cong\;& 0.468\%.\end{aligned}$$

5. (a) Settlement date = 20/12/11 + 5 dias de calendário = 23/12/11.
Pretende-se avaliar uma obrigação com os seguintes cash flows vincendos:



Portanto,

$$\begin{aligned}B_0 &= 3\%P(0,0.4153)+3\%P(0,1.4153)+103\%P(0,2.4153)\\ &= 3\%P(0,0.4153)+3\%\times 0.9820+103\%\times 0.9663.\end{aligned} \tag{24}$$

Relativamente ao factor de desconto a 0.4153 anos, via equações (346), (343) e (344) dos apontamentos,

$$\gamma=\sqrt{(0.5)^2+2\times(0.05)^2}\cong 0.5050,$$

$$\begin{aligned}B(0.4153) &= -\frac{2\times\left(e^{0.505\times 0.4153}-1\right)}{2\times 0.505+(0.5+0.505)\times\left(e^{0.505\times 0.4153}-1\right)}\\ &\cong -0.3750,\end{aligned}$$

e

$$\begin{aligned}A(0.4153) &= \frac{2\times 0.5\times 0.02}{(0.05)^2}\times\ln\left[\frac{2\times 0.505\times e^{(0.5+0.505)\times\frac{0.4153}{2}}}{2\times 0.505+(0.5+0.505)\times\left(e^{0.505\times 0.4153}-1\right)}\right]\\ &\cong -0.0008.\end{aligned}$$

Portanto,

$$\begin{aligned} P(0, 0.4153) &= \exp(-0.0008 - 0.3750 \times 1\%) \\ &\cong 0.9955. \end{aligned}$$

Retomando a equação (24), então

$$\begin{aligned} B_0 &= 3\% \times 0.9955 + 3\% \times 0.9820 + 103\% \times 0.9663 \\ &\cong 105.464\%. \end{aligned}$$

(b)

$$\begin{aligned} VT^{bid} &= \frac{3\%}{(1+1.47\%)^{0.4153}} + \frac{3\%}{(1+1.47\%)^{1.4153}} + \frac{103\%}{(1+1.47\%)^{2.4153}} \\ &\cong 105.35\%. \end{aligned}$$

Visto que $VT^{bid} < B_0$, então decide-se não vender.

$$\begin{aligned} VT^{ask} &= \frac{3\%}{(1+1.449\%)^{0.4153}} + \frac{3\%}{(1+1.449\%)^{1.4153}} + \frac{103\%}{(1+1.449\%)^{2.4153}} \\ &\cong 105.40\%. \end{aligned}$$

Visto que $VT^{ask} < B_0$, então decide-se comprar.

(c) Seja $t = 0$, $T = 2$ anos, $K = 98.50\%$, $M = \$1M$ e $P(T, T+1)$ o factor de desconto a 12 meses em vigor daqui a 2 anos. Consequentemente, o payoff terminal da opção, daqui a 2 anos, é dado por

$$V_T = M \times P(T, T+1) \times 1\!\!1_{\{P(T,T+1)<K\}}.$$

Consequentemente, o valor actual da opção é dado por

$$\begin{aligned} V_t &= P(t, T+1) \times \mathbb{E}_{\mathbb{Q}_{T+1}} \left[\left. \frac{M \times P(T, T+1) \times 1\!\!1_{\{P(T,T+1)<K\}}}{P(T, T+1)} \right| \mathcal{F}_t \right] \\ &= M \times P(t, T+1) \times \mathbb{E}_{\mathbb{Q}_{T+1}} \left[\left. 1\!\!1_{\{P(T,T+1)<K\}} \right| \mathcal{F}_t \right] \\ &= M \times P(t, T+1) \times \mathbb{Q}_{T+1} \left[P(T, T+1) < K \right| \mathcal{F}_t]. \end{aligned}$$

Visto que

$$P(T, T+1) = \exp[A(1) + B(1) r_T],$$

então

$$\begin{aligned} V_t &= M \times P(t, T+1) \times \mathbb{Q}_{T+1} [A(1) + B(1) r_T < \ln(K) | \mathcal{F}_t] \\ &= M \times P(t, T+1) \times \mathbb{Q}_{T+1} \left[r_T > \left. \frac{\ln(K) - A(1)}{B(1)} \right| \mathcal{F}_t \right] \\ &= M \times P(t, T+1) \times Q_{\chi^2\left(\frac{4k\theta}{\sigma^2}, \zeta\right)} \left[\frac{\ln(K) - A(1)}{B(1) L} \right], \end{aligned} \tag{25}$$

visto que

$$\frac{r_T}{L} \overset{\mathbb{Q}_{T+1}}{\sim} \chi^2\left(\frac{4k\theta}{\sigma^2}, \zeta\right),$$

com

$$\begin{aligned}
\zeta &= \frac{8r_t\gamma^2 e^{\gamma(T-t)}}{\sigma^2\left[e^{\gamma(T-t)}-1\right]\left\{\gamma\left[e^{\gamma(T-t)}+1\right]+\left[k-\sigma^2\times B(1)\right]\left[e^{\gamma(T-t)}-1\right]\right\}} \\
&= \zeta_2 \\
&= \frac{(0.05)^{-2}\left(e^{0.505\times 2}-1\right)^{-1}\left[8\times 1\%\times(0.505)^2\times e^{0.505\times 2}\right]}{\left\{0.505\left[e^{0.505\times 2}+1\right]+\left[0.5-(0.05)^2(-0.7867)\right]\left[e^{0.505\times 2}-1\right]\right\}} \\
&\cong 4.637694995
\end{aligned}$$

e

$$\begin{aligned}
L &= \frac{\sigma^2}{2}\frac{e^{\gamma(T-t)}-1}{\gamma\left[e^{\gamma(T-t)}+1\right]+\left[k-\sigma^2\times B(1)\right]\left[e^{\gamma(T-t)}-1\right]} \\
&= L_2 \\
&= \frac{(0.05)^2}{2}\frac{e^{0.505\times 2}-1}{0.505\times\left[e^{0.505\times 2}+1\right]+\left[0.5-(0.05)^2\times(-0.7867)\right]\left[e^{0.505\times 2}-1\right]} \\
&\cong 0.000788367.
\end{aligned}$$

Portanto,

$$\begin{aligned}
& Q_{\chi^2\left(\frac{4k\theta}{\sigma^2},\zeta\right)}\left[\frac{\ln(K)-A(1)}{B(1)\,L}\right] \\
&= Q_{\chi^2\left(\frac{4\times 0.5\times 2\%}{0.05^2},4.637694995\right)}\left[\frac{\ln(98.5\%)-(-0.0043)}{-0.7867\times 0.000788367}\right] \\
&= Q_{\chi^2(16,4.637694995)}(17.49952893) \\
&\cong 0.361963928.
\end{aligned}$$

Finalmente, recuperando a equação (25), então

$$\begin{aligned}
V_t &= \$1M\times 0.9566\times 0.361963928 \\
&\cong EUR346,242.03.
\end{aligned}$$