Modelos de Estrutura Temporal de Taxas de Juro
Mestrado em Matemática Financeira 15/16
IBS e FCUL

Exame 1ª Época

21/Dez/16 Duração: 3h

Case 1 Please answer only two of the following questions: (2x2.5V)

a) Under the Cox, Ingersoll and Ross (1985) model, compute the (time t) *fair value* of a *cash-or-nothing put* with expiry date at time $T+\delta$, with a *contract size* equal to M, with a *strike* equal to k, and on the Euribor rate $E(T, T+\delta)$ to prevail between times T $(\geq t)$ and $T+\delta$ (with $\delta > 0$). For this purpose, consider that $P(T, T+\delta) = [1+\delta \times E(T, T+\delta)]^{-1}$.

b) Under the Vasiček (1977) model, compute the (time t) *fair value* of an *asset-or-nothing call* with expiry date at time T, with a *contract size* equal to M, with a *strike* equal to K, and on a zero-coupon bond with expiry date at time $T+\delta$ (with $\delta > 0$).

c) Under a CEV model with $\beta > 2$, compute the (time t) *fair value* of an *asset-or-nothing put* with expiry date at time T, with a *contract size* equal to M, with a *strike* equal to K, and on the stock S.

Case 2 Consider a CEV process given by the following SDE:

$$dS_t = (r-q)\, S_t dt + \delta S_t^{\frac{\beta}{2}} dW_t^{\mathbb{Q}}.$$

Assume that $S = \$100$, $\beta = -3$, $r = 1\%$, $q = 3\%$ and that the (annualized) standard deviation of stock returns is equal to 30%. Please consider also the following table containing cumulative probabilities associated to a noncentral chi-square random variable with 2.4 degrees of freedom and a noncentrality parameter equal to 3.467407:

x	0.40000	2.40000	3.64519
F(x)	0.02811	0.22084	0.36183

Please answer the following questions:

a) Price an ATM European-style standard put on the stock S and with a time-to-maturity of 0.5 years. (2V)

b) Price an ATM European-style asset-or-nothing put on the stock S, with a time-to-maturity of 0.5 years, and with a contract size of 100 shares. (1V)

Case 3 Consider the following parameters for the Heston (1993) model:

- Spot price of the EVN stock = EUR10;

- *Dividend yield* for the EVN stock (continuous compounding) = 3% (30/360);
- Risk-free interest rate (continuous compounding) = 1% (30/360);
- Instantaneous variance of the stock returns (v) = 0.08;
- Speed of mean reversion of the volatility (k) = 4;
- Long-term level of the instantaneous variance (θ) = 0.09;
- Volatility of the instantaneous variance (σ) = 10%; and
- Correlation coefficient between the stock price and the instantaneous variance (ρ) = -0.6.

Next table summarizes the implementation of equations (173) and (174) of the handouts for the strikes EUR10 and EUR12, for a maturity of 3 months, and through a Gauss-Laguerre quadrature with 15 nodes:

w_i	ϕ_i	$X = 10$		$X = 12$	
		$f_1(\phi_i)$	$f_2(\phi_i)$	$f_1(\phi_i)$	$f_2(\phi_i)$
2.1823E-01	9.3308E-02	5.9314E-03	-1.6970E-02	-1.9419E-01	-2.1709E-01
3.4221E-01	4.9269E-01	8.8431E-03	-2.5216E-02	-2.8845E-01	-3.2235E-01
2.6303E-01	1.2156E+00	1.8218E-02	-5.1044E-02	-5.8278E-01	-6.5011E-01
1.2643E-01	2.2699E+00	5.2220E-02	-1.3898E-01	-1.5778E+00	-1.7506E+00
4.0207E-02	3.6676E+00	2.0967E-01	-4.9888E-01	-5.5937E+00	-6.1296E+00
8.5639E-03	5.4253E+00	1.1799E+00	-2.2916E+00	-2.5136E+01	-2.6862E+01
1.2124E-03	7.5659E+00	9.1817E+00	-1.2865E+01	-1.3678E+02	-1.3935E+02
1.1167E-04	1.0120E+01	9.5708E+01	-8.1304E+01	-8.4384E+02	-7.8318E+02
6.4599E-06	1.3130E+01	1.2733E+03	-4.6731E+02	-5.2973E+03	-3.9680E+03
2.2263E-07	1.6654E+01	2.0323E+04	2.6466E+02	-2.6184E+04	-6.3218E+03
4.2274E-09	2.0776E+01	3.5920E+05	1.0762E+05	3.0885E+04	2.6654E+05
3.9219E-11	2.5624E+01	6.2026E+06	3.3014E+06	3.4255E+06	5.4638E+06
1.4565E-13	3.1408E+01	7.8786E+07	6.4306E+07	5.7866E+07	6.8380E+07
1.4830E-16	3.8531E+01	4.0795E+07	4.4581E+08	5.7338E+08	6.4875E+08
1.6006E-20	4.8026E+01	-4.1462E+09	-4.6017E+09	3.6447E+08	3.5530E+09
	$\sum_{i=1}^{15} w_i f_j(\phi_i) =$	7.0593E-02	-0.11005532	-1.23387061	-1.30859885

Please answer the following questions:

a) Price a European-style call on the GN stock, with a strike price equal to EUR10, and with a time-to-maturity of 0.25 years. (1V)

b) Price a *range asset-or-nothing option* on the GN stock, with a time-to-maturity of 0.25 years, a contract size equal to 100 shares, and *strikes* equal to EUR10 and EUR12. (1V)

Case 4 Consider the following parameters, estimated under measure $\mathbb{Q}$, for the Vasiček (1977) model, using German treasury bonds for the settlement date of 21/12/16:

alpha	2
gamma	1%
rho	8%
r(t)	0.5%

Next table shows discount factors (for different maturities) based on the previous model' parameters:

T-t	B(t,T)	A(t,T)	P(t,T)
0.5	0.3161	-0.0018	0.9967
1	0.4323	-0.0054	0.9925
1.3014	0.4630	-0.0079	0.9899
2	0.4908	-0.0141	0.9836
2.3014	0.4950	-0.0168	0.9809
3	0.4988	-0.0232	0.9746

The market also trades European-style options with a time-to-maturity of 0.3014 years, and on German treasury zero-coupon bonds with a time-to-maturity of 2.3014 years:

strike	98.119%	99.035%
Call	0.722%	0.335%
Put	0.569%	1.096%

Please answer the following questions:

a) Find the fair value of a German treasury bond with maturity at 10/04/2019, with *bullet* redemption, and with a coupon rate of 3% (annual coupon under the day-count convention ACT/ACT). The first (short) coupon will be paid on 10/04/2017 but the first coupon period started on 21/11/2016. Consider that the number of calendar days between 21/11/2016 and 21/12/2016 is equal to 30 days, and the number of calendar days between 21/12/16 and 10/04/2017 is equal to 110 days. (1.5V)

b) Formulate a trading decision, knowing that the German treasury bond is quoted at a clean price of 104.95%(bid)-105.00%(ask). (0.5V)

c) Price a European-style put, with maturity at 10/04/2017, with a strike price of 99.035%, and on a Treasury Bill with maturity at 10/04/2018. (2V)

d) Price a European-style put with a maturity at 10/04/2017, with a strike equal to 104.03%, and on the coupon-bearing bond described in question a). For this purpose, consider that an instantaneous interest rate of 1% yields, on 10/04/2017, a fair value of 104.03% for the underlying coupon-bearing bond. (2V)

Caso 5 Consider the following parameters, estimated under measure $\mathbb{Q}$ and using EUR IRS quotes, for the Cox et al. (1985):

k	2.0
theta	1.0%
sigma	8.0%
r	0.5%

Next table contains discount factors (for different maturities) based on the previous model' parameters:

T-t	B(T-t)	A(T-t)	P(t,T)
0.5	-0.3160	-0.0018	0.9966
1	-0.4322	-0.0057	0.9922
1.5	-0.4748	-0.0102	0.9875
2	-0.4905	-0.0151	0.9826
2.5	-0.4963	-0.0200	0.9777
3	-0.4984	-0.0250	0.9729

Consider also the following table containing cumulative probabilities associated to a noncentral chi-square random variable with 12.5 degrees of freedom and a noncentrality parameter equal to b:

	F(x)		
x	12.5000000	13.4725409	13.4805925
b			
0.97627458	4.78600E-01	5.52401E-01	5.52993E-01
0.97685803	4.78557E-01	5.52358E-01	5.52950E-01

Please answer the following questions:

a) Find the fixed interest rate for a 3-years interest rate swap starting today, with annual compounding, and quoted against the 12-month Euribor. (2V)

b) Price a European-style *at-the-money forward* call with a time-to-maturity of one year, and on a zero-coupon bond with a time-to-maturity of two years, knowing that $L_1 = 0.00069145$ and $\zeta_1 = 0.97685803$. (2V)

Referências

Cox, J., J. Ingersoll, and S. Ross, 1985, A Theory of the Term Structure of Interest Rates, *Econometrica* 53, 385–407.

Heston, S., 1993, A Closed-Form Solution for Options with Stochastic Volatility with Applications to Bond and Currency Options, *Review of Financial Studies* 6, 327–343.

Vasiček, O., 1977, An Equilibrium Characterization of the Term Structure, *Journal of Financial Economics* 5, 177–188.