OPÇÕES EXÓTICAS
MSc MATEMÁTICA FINANCEIRA 2009/10
EXAME - Resolução

29/07/10 **Duração: 2.5 horas**

CASO 1

a) Formule, no momento "t" ($\leq T$), o cálculo do fair value de um depósito com vencimento no momento "T", e que se valoriza à taxa de juro α (em regime de capitalização contínua) caso o índice CAC40 não ultrapasse o nível "U" até momento "T"; caso contrário, i.e. caso o índice CAC40 ultrapasse o nível "U" até momento "T", então o depósito será capitalizado à taxa de juro α (em regime de capitalização contínua) até à data em que o nível "U" é alcançado e à taxa de juro $\beta(>\alpha)$ (em regime de capitalização contínua) desde essa data e até ao momento "T".

O valor actual do depósito é igual a:

$$B_0 = 100\% \times e^{-r(T-t)} + RV_0,$$

Sendo "r" a taxa de juro sem risco (em RCC) e o valor terminal da remuneração variável dado por

$$RV_T = 100\% \times \begin{cases} e^{\alpha(v-t)} e^{\beta(T-v)} \Leftarrow v \leq T \\ e^{\alpha(T-t)} \Leftarrow v > T \end{cases},$$

sendo "v" o first passage time do índice "S" pelo nível "U", i.e.

$$v = \inf\{u > t : S_u = U\}.$$

Utilizando risk-neutral valuation,

$$\begin{aligned} RV_0 &= 100\% \times e^{-r(T-t)} E_Q\left(e^{\alpha(T-t)} 1_{\{v>T\}} \mid F_t\right) + 100\% \times e^{-r(T-t)} E_Q\left(e^{\alpha(v-t)} e^{\beta(T-v)} 1_{\{v\leq T\}} \mid F_t\right) \\ &= 100\% \times e^{(\alpha-r)(T-t)} Q\left(v > T \mid F_t\right) + 100\% \times e^{-r(T-t)-\alpha t+\beta T} E_Q\left(e^{v(\alpha-\beta)} 1_{\{v\leq T\}} \mid F_t\right). \end{aligned} \tag{1}$$

Relativamente ao 1º termo da equação (1), assumindo que a cotação do índice segue um GBM e utilizando a Proposição 45 com $\eta = 1$,

$$Q(v > T \mid F_t) = Q\left(\sup_{t<u<T}(S_u) < U \mid F_t\right)$$

$$= Q\left[\sup_{t<u<T}\left(\ln\left(\frac{S_u}{S_t}\right)\right) < \ln\left(\frac{U}{S_t}\right) \mid F_t\right] \tag{2}$$

$$= \Phi\left[-d_2^M(S_t, U)\right] - \left(\frac{U}{S_t}\right)^{\frac{2\mu}{\sigma^2}} \Phi\left[-d_2^M(U, S_t)\right],$$

sendo $\mu = r - q - \sigma^2/2$.

Relativamente ao 2º termo da equação (1), e utilizando a Proposição 47 com $\eta = 1$,

$$E_Q\left(e^{v(\alpha-\beta)} 1_{\{v \le T\}} \mid F_t\right)$$

$$= \int_t^T e^{v(\alpha-\beta)} \frac{\ln\left(\frac{U}{S_t}\right)}{\sigma^2\sqrt{(v-t)^3}} \exp\left\{-\frac{\left[\ln\left(\frac{U}{S_t}\right) - \mu(v-t)\right]^2}{2\sigma^2(v-t)}\right\} dv$$

$$= e^{t(\alpha-\beta)} \int_t^T e^{(v-t)(\alpha-\beta)} \frac{\ln\left(\frac{U}{S_t}\right)}{\sigma^2\sqrt{(v-t)^3}} \exp\left\{-\frac{\left[\ln\left(\frac{U}{S_t}\right) - \mu(v-t)\right]^2}{2\sigma^2(v-t)}\right\} dv$$

I.e.

$$E_Q\left(e^{v(\alpha-\beta)} 1_{\{v \le T\}} \mid F_t\right)$$

$$= e^{t(\alpha-\beta)} \ln\left(\frac{U}{S_t}\right) \int_t^T \frac{1}{\sigma^2\sqrt{(v-t)^3}} \exp\left\{(v-t)(\alpha-\beta) - \frac{\left[\ln\left(\frac{U}{S_t}\right)\right]^2 - 2\ln\left(\frac{U}{S_t}\right)\mu(v-t) + \mu^2(v-t)^2}{2\sigma^2(v-t)}\right\} dv$$

$$E_Q\left(e^{v(\alpha-\beta)}1_{\{v\le T\}} \mid F_t\right)$$

$$= e^{t(\alpha-\beta)}\ln\left(\frac{U}{S_t}\right)\int_t^T \frac{1}{\sigma^2\sqrt{(v-t)^3}}\exp\left\{(v-t)(\alpha-\beta)-\frac{\left[\ln\left(\frac{U}{S_t}\right)\right]^2}{2\sigma^2(v-t)}+\frac{\ln\left(\frac{U}{S_t}\right)\mu}{\sigma^2}-\frac{\mu^2(v-t)}{2\sigma^2}\right\}dv$$

$$= e^{t(\alpha-\beta)}\ln\left(\frac{U}{S_t}\right)\exp\left[\frac{\mu}{\sigma^2}\ln\left(\frac{U}{S_t}\right)\right]\int_t^T \frac{1}{\sigma^2\sqrt{(v-t)^3}}\exp\left\{-\frac{\left[\ln\left(\frac{U}{S_t}\right)\right]^2}{2\sigma^2(v-t)}-\frac{\left(\mu^2-2\sigma^2(\alpha-\beta)\right)(v-t)}{2\sigma^2}\right\}dv$$

$$= e^{t(\alpha-\beta)}\ln\left(\frac{U}{S_t}\right)\exp\left[\frac{\mu}{\sigma^2}\ln\left(\frac{U}{S_t}\right)-\frac{2\ln\left(\frac{U}{S_t}\right)\sqrt{\mu^2-2\sigma^2(\alpha-\beta)}(v-t)}{2\sigma^2(v-t)}\right]$$

$$\int_t^T \frac{1}{\sigma^2\sqrt{(v-t)^3}}\exp\left\{-\frac{\left[\ln\left(\frac{U}{S_t}\right)\right]^2+\left(\mu^2-2\sigma^2(\alpha-\beta)\right)(v-t)^2-2\ln\left(\frac{U}{S_t}\right)\sqrt{\mu^2-2\sigma^2(\alpha-\beta)}(v-t)}{2\sigma^2(v-t)}\right\}dv$$

$$= e^{t(\alpha-\beta)}\exp\left[\frac{\mu}{\sigma^2}\ln\left(\frac{U}{S_t}\right)-\frac{\overline{\psi}}{\sigma^2}\ln\left(\frac{U}{S_t}\right)\right]\int_t^T \frac{\ln\left(\frac{U}{S_t}\right)}{\sigma^2\sqrt{(v-t)^3}}\exp\left\{-\frac{\left[\ln\left(\frac{U}{S_t}\right)-\overline{\psi}(v-t)\right]^2}{2\sigma^2(v-t)}\right\}dv,$$

sendo $\overline{\psi} := \sqrt{\mu^2-2\sigma^2(\alpha-\beta)}$.

Via Proposição 47,

$$E_Q\left(e^{v(\alpha-\beta)}1_{\{v\le T\}} \mid F_t\right)$$

$$= e^{t(\alpha-\beta)}\left(\frac{U}{S_t}\right)^{\frac{\mu-\overline{\psi}}{\sigma^2}}\int_t^T Q\left[\overline{X}_v=\ln\left(\frac{U}{S_t}\right),\sup_{t\le u<T}\left(\overline{X}_u\right)<\ln\left(\frac{U}{S_t}\right)\mid F_t\right]dv,$$

sendo $d\overline{X}_t = \overline{\psi}dt+\sigma dW_t^Q$.

Consequentemente,

$$E_Q\left(e^{v(\alpha-\beta)}1_{\{v\leq T\}} \mid F_t\right)=e^{t(\alpha-\beta)}\left(\frac{U}{S_t}\right)^{\frac{\mu-\overline{\psi}}{\sigma^2}} Q\left(\overline{v}<T \mid F_t\right),$$

sendo

$$\overline{v}=\inf\left\{u>t : \overline{X}_u = \ln\left(\frac{U}{S_t}\right)\right\}.$$

Utilizando, por exemplo, a equação (130) dos apontamentos com $\mu=\overline{\psi}$,

$$E_Q\left(e^{v(\alpha-\beta)}1_{\{v\leq T\}} \mid F_t\right)$$

$$=e^{t(\alpha-\beta)}\left(\frac{U}{S_t}\right)^{\frac{\mu-\overline{\psi}}{\sigma^2}}\left\{\Phi\left[-\frac{\ln\left(\frac{U}{S_t}\right)-\overline{\psi}(T-t)}{\sigma\sqrt{T-t}}\right]+\left(\frac{U}{S_t}\right)^{\frac{2\overline{\psi}}{\sigma^2}}\Phi\left[\frac{-\ln\left(\frac{U}{S_t}\right)-\overline{\psi}(T-t)}{\sigma\sqrt{T-t}}\right]\right\}. \quad (3)$$

Em suma, combinando as equações (1), (2) e (3),

$$RV_0=100\%\times e^{(\alpha-r)(T-t)}\left\{\Phi\left[-d_2^M(S_t,U)\right]-\left(\frac{U}{S_t}\right)^{\frac{2\mu}{\sigma^2}}\Phi\left[-d_2^M(U,S_t)\right]\right\}$$

$$+100\%\times e^{(\beta-r)(T-t)}\left\{\left(\frac{U}{S_t}\right)^{\frac{\mu-\overline{\psi}}{\sigma^2}}\Phi\left[-\frac{\ln\left(\frac{U}{S_t}\right)-\overline{\psi}(T-t)}{\sigma\sqrt{T-t}}\right]+\left(\frac{U}{S_t}\right)^{\frac{\mu+\overline{\psi}}{\sigma^2}}\Phi\left[\frac{\ln\left(\frac{S_t}{U}\right)-\overline{\psi}(T-t)}{\sigma\sqrt{T-t}}\right]\right\}.$$

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c) Deduza a fórmula de avaliação de uma *European-style down-and-out call* com uma barreira (L) superior ao *strike* (X).

Via página 35 dos apontamentos,

$DO(1)_t\,(S;X;L>X;R=0;T)$

$$= e^{-r\tau}\int_{\ln\left(\frac{L}{S_t}\right)}^{\infty}\left(S_t e^{y}-X\right)\left\{\phi\left(y;\mu\tau,\sigma\sqrt{\tau}\right)-\left(\frac{L}{S_t}\right)^{\frac{2\mu}{\sigma^2}}\phi\left[y;2\ln\left(\frac{L}{S_t}\right)+\mu\tau,\sigma\sqrt{\tau}\right]\right\}dy.$$

Fazendo a mudança de variável de integração $z=y-2\ln\left(\frac{L}{S_t}\right)$,

$DO(1)_t\,(S;X;L>X;R=0;T)$

$$= e^{-r\tau}\int_{\ln\left(\frac{L}{S_t}\right)}^{\infty}\left[S_t e^{y}-L+(L-X)\right]\phi\left(y;\mu\tau,\sigma\sqrt{\tau}\right)dy$$

$$-\left(\frac{L}{S_t}\right)^{\frac{2\mu}{\sigma^2}} e^{-r\tau}\int_{\ln\left(\frac{L}{L^2/S_t}\right)}^{\infty}\left[S_t\left(\frac{L}{S_t}\right)^{2}e^{z}-L+(L-X)\right]\phi\left(z;\mu\tau,\sigma\sqrt{\tau}\right)dz$$

$$= c_t(S;L;T)+e^{-r\tau}(L-X)\Phi\left[-\frac{\ln\left(\frac{L}{S_t}\right)-\mu\tau}{\sigma\sqrt{\tau}}\right]$$

$$-\left(\frac{L}{S_t}\right)^{\frac{2\mu}{\sigma^2}}\left\{c_t\left(L^2/S_t;L;T\right)+e^{-r\tau}(L-X)\Phi\left[-\frac{\ln\left(\frac{S_t}{L}\right)-\mu\tau}{\sigma\sqrt{\tau}}\right]\right\}.$$

CASO 2

a)

$$r:\quad e^{r\times 6/12}=1+1.2\%\times 6/12 \Rightarrow r=\frac{12}{6}\ln\left(1+1.2\%\times 6/12\right)\cong 1.196\%.$$

$$\varepsilon_{1,8}: \quad 5{,}236.93 = 6{,}000 \times \exp\left\{\left[1.196\% - 1.799\% - \frac{(0.3)^2}{2}\right] \times \frac{1}{12} + \varepsilon_{1,8} \times 0.3 \times \sqrt{\frac{1}{12}}\right\}$$

$\varepsilon_{1,8} \cong$ -1.5216.

$$S_{5,2} = 4{,}787.88 \times \exp\left\{\left[1.196\% - 1.799\% - \frac{(0.3)^2}{2}\right] \times \frac{1}{12} + (-0.6902) \times 0.3 \times \sqrt{\frac{1}{12}}\right\}$$

$S_{5,2} \cong$ 4,490.94.

$$\min_{i=1,\dots,6}\left(S_{i,12}\right) = 5{,}564.33 < 5{,}900 \Rightarrow \quad V_{6,12} = \max(6{,}000 - 7{,}263.89\ ; 0) = 0.$$

$$\min_{i=1,\dots,6}\left(S_{i,13}\right) = 4{,}773.24 < 5{,}900 \Rightarrow \quad V_{6,13} = \max(6{,}000 - 4{,}773.24\ ; 0) = 1{,}226.76.$$

b)

$$\hat{V}_0 = e^{-1.196\% \times 0.5} \times \frac{11{,}884.92}{20} \cong 590.70.$$

$$\sigma\left(\hat{V}_0\right) = \frac{e^{-1.196\% \times 0.5}}{\sqrt{20}} \times \sqrt{\frac{22{,}359{,}524.14 - {(11{,}884.92)^2}/{20}}{20-1}} \cong 199.44 .$$

c)

Valor actual do depósito bancário:

$$B_0 = \frac{105\%}{1 + 1.2\% \times {6}/{12}} + RV_0.$$

Por seu turno,

$\max(S_{i,j}) \leq 7{,}000 \wedge \min(S_{i,j}) \geq 5{,}500$

$$RV_{6M,j} = \begin{cases} 60\% \times \max\left(\dfrac{S_{6M} - 6{,}000}{6{,}000}; 0\%\right) \Leftarrow \exists i \in \{1,2,...,6\} : \max(S_{i,j}) \leq 7{,}000 \wedge \min(S_{i,j}) \geq 5{,}500 \\ 0\% \Leftarrow Else \end{cases}$$

$$= \frac{60\%}{6{,}000} \times \begin{cases} \max(S_{6M} - 6{,}000; 0) \Leftarrow \exists i \in \{1,2,...,6\} : \max(S_{i,j}) \leq 7{,}000 \wedge \min(S_{i,j}) \geq 5{,}500 \\ 0 \Leftarrow Else \end{cases}$$

Portanto, a remuneração variável envolve uma double-barrier knock-out call com zero rebate (V):

$$RV_0 = \frac{60\%}{6{,}000} \times V_0.$$

O quadro seguinte resume a avaliação da anterior double-barrier knock-out call:

Hl	5500	
Hu	7000	

j	Exit (0) Inside (1)	$V_{6M,j}$
1	0	-
2	0	-
3	0	-
4	0	-
5	0	-
6	0	-
7	0	-
8	0	-
9	0	-
10	0	-
11	1	121.02
12	0	-
13	0	-
14	0	-
15	0	-
16	1	475.73
17	0	-
18	1	-
19	0	-
20	0	-
	total	596.75

$$RV_0 = \frac{60\%}{6{,}000} \times \left(\frac{596.75}{20} \times e^{-1.196\% \times 0.5} \right)$$

$$= \frac{60\%}{6{,}000} \times 29.66$$

$$\cong 0.30\%.$$

Em suma,

$$B_0 = 104.37\% + 0.30\% = 104.67\%.$$

<u>CASO 3</u>

a)

$$r: \quad e^{r \times 6/12} = 1 + 1\% \times 6/12 \Rightarrow r = \frac{12}{6} \ln\left(1 + 1\% \times 6/12\right) \cong 0.998\%.$$

$$B_0 = \frac{100\%}{1 + 1\% \times 6/12} + RV_0.$$

$$RV_{6M} = \begin{cases} 8\% \Leftarrow 3{,}000 \times 90\% = 2{,}700 < S_{6M} < 3{,}000 \\ 0\% \Leftarrow ELSE \end{cases}$$

$$= 8\% \times RD_{6M}\left(M = 1; S; X_a = 2{,}700; X_a = 3{,}000; T = 6M\right).$$

Portanto,

$$RV_0 = 8\% \times RD_0\left(M = 1; S; X_a = 2{,}700; X_a = 3{,}000; T = 6M\right)$$
$$= 8\% \times e^{-0.998\% \times 0.5} \times \left\{N\left[d_2^M(2{,}700)\right] - N\left[d_2^M(3{,}000)\right]\right\}.$$

$$\Phi\left[d_2^M(2{,}700)\right] = \Phi\left[\frac{\ln\left(3{,}000/2{,}700\right) + \left(0.998\% - 2\% - (0.2)^2/2\right) \times 0.5}{0.2 \times \sqrt{0.5}}\right]$$

$$\cong \Phi(0.6389) = 0.7385.$$

$$\Phi\left[d_2^M(3{,}000)\right] = \Phi\left[\frac{\ln\left(3{,}000/3{,}000\right) + \left(0.998\% - 2\% - (0.2)^2/2\right)\times 0.5}{0.2\times\sqrt{0.5}}\right]$$

$$\cong \Phi(-0.1062) = 0.4577.$$

$\underline{RV_0 = 8\% \times e^{-0.998\% \times 0.5} \times (0.7385 - 0.4577) = 2.24\%.}$

$B_0 = 99.50\% + 2.24\% \cong 101.74\% > 100\% \Rightarrow$ Investir.

b)

Via proposição 20 e equação (58):

$$p_0\left[p_0(S_0; X = 3{,}000; 1y); 361.57; 0.5y\right]$$

$$= 3{,}000 \times e^{-2\% \times 1} \times M\left(a_1^{**}, -b_1; -\sqrt{\frac{0.5}{1}}\right) - 3{,}000 \times e^{-0.998\% \times 1} \times M\left(a_2^{**}, -b_2; -\sqrt{0.5}\right)$$

$$+ 361.57 \times e^{-0.998\% \times 0.5} \times \Phi\left(a_2^{**}\right).$$

Visto que:

$S^{**} = 2{,}700;$

$$a_1^{**} = \frac{\ln\left(\frac{3{,}000}{2{,}700}\right) + \left(0.998\% - 2\% + \frac{(0.2)^2}{2}\right)\times 0.5}{0.2\times\sqrt{0.5}} \cong 0.78029576;$$

$$a_2^{**} = 0.78029576 - 0.2\times\sqrt{0.5} \cong 0.6389;$$

$$b_1 = \frac{\ln\left(\frac{3{,}000}{3{,}000}\right) + \left(0.998\% - 2\% + \frac{(0.2)^2}{2}\right)\times 1}{0.2\times\sqrt{1}} \cong 0.04987542;$$

$$b_2 = 0.04987542 - 0.2\times\sqrt{1} \cong \text{-}0.15012;$$

então:

$$p_0\left[p_0\left(S_0;X=3{,}000;1y\right);361.57;0.5y\right]$$

$$=3{,}000\times e^{-2\%\times 1}\times M\left(0.780296,-0.049875;-0.707107\right)$$

$$-3{,}000\times e^{-0.998\%\times 1}\times M\left(0.6389,0.15012;-0.707107\right)$$

$$+361.57\times e^{-0.998\%\times 0.5}\times\Phi\left(0.6389\right).$$

Utilizando a tabela de probabilidades,

$$p_0\left[p_0\left(S_0;X=3{,}000;1y\right);361.57;0.5y\right]$$

$$=3{,}000\times e^{-2\%\times 1}\times 0.28897-3{,}000\times e^{-0.998\%\times 1}\times 0.322596+361.57\times e^{-0.998\%\times 0.5}\times 0.738548$$

$$\cong 157.30.$$

c)

$$B_0=\frac{100\%}{1+1\%\times {}^{6}\!/_{12}}+RV_0.$$

$$RV_{6M}=\begin{cases}40\%\times\max\left(0\%;\dfrac{S_{6M}-S_0}{S_0}\right)\Leftarrow \inf\limits_{0<u\le 0.5}\left(S_u\right)\le 2{,}975.31\\ 0.5\%\Leftarrow \inf\limits_{0<u\le 0.5}\left(S_u\right)>2{,}975.31\end{cases}$$

$$=\frac{40\%}{S_0}\times\begin{cases}\max\left(0\%;S_{6M}-S_0\right)\Leftarrow \inf\limits_{0<u\le 0.5}\left(S_u\right)\le 2{,}975.31\\ 0.5\%\times\dfrac{S_0}{40\%}\Leftarrow \inf\limits_{0<u\le 0.5}\left(S_u\right)>2{,}975.31\end{cases}$$

$$=\frac{40\%}{S_0}\times DI(1)_{0.5}\left(S_{6M};X=S_0;L=2{,}975.31;R=0.0125S_0;T=0.5y\right).$$

Portanto,

$$RV_0 = \frac{40\%}{S_0} \times DI(1)_0 (S_0; X = 3{,}000; L = 2{,}975.31; R = 0.0125 \times 3{,}000; T = 0.5y)$$

Por outro lado,

$$DI(1)_0 (S_0; X = 3{,}000; L = 2{,}975.31; R = 37.5; T = 0.5y)$$

$$= DI(1)_0 (S_0; X = 3{,}000; L = 2{,}975.31; R = 0; T = 0.5y) + KIR(-1)_0,$$

onde $KIR(-1)_0$ designa o valor actual de um deferrable knock-in rebate com down barrier.

Começando pela down-and-in call sem rebate, utilizando a Proposição 37, e visto que L<X, então:

$$DI(1)_0 (S_0; X = 3{,}000; L = 2{,}975.31; R = 0; T = 0.5y) + KIR(-1)_0$$

$$= \left(\frac{2{,}975.31}{3{,}000} \right)^{\frac{2\mu}{\sigma^2}} c_0 \left(\frac{(2{,}975.31)^2}{3{,}000}; 3{,}000; T = 0.5y \right)$$

$$= \left(\frac{2{,}975.31}{3{,}000} \right)^{\frac{2\mu}{\sigma^2}} \frac{(2{,}975.31)^2}{(3{,}000)^2} c_0 \left(3{,}000; 3{,}000 \times \frac{(3{,}000)^2}{(2{,}975.31)^2} \cong 3{,}050; T = 0.5y \right).$$

Visto que

$$\mu = 0.998\% - 2\% - {(0.2)^2}\big/{2} \cong \text{-}0.03002,$$

e utilizando os dados do enunciado, então

$$DI(1)_0 (S_0; X = 3{,}000; L = 2{,}975.31; R = 0; T = 0.5y) + KIR(-1)_0$$

$$= \left(\frac{2{,}975.31}{3{,}000} \right)^{\frac{2\times(\text{-}0.03002)}{(0.2)^2}} \times \frac{(2{,}975.31)^2}{(3{,}000)^2} \times 138.87 \cong 138.30.$$

Relativamente ao rebate, e utilizando a Proposição 45,

$KIR(-1)_0$

$$= \frac{37.5}{1+1\% \times 6/12} \times \left\{ \Phi\left[d_2^M (3{,}000; 2{,}975.31) \right] - \left(\frac{2{,}975.31}{3{,}000} \right)^{\frac{2\times(-0.03002)}{0.2^2}} \Phi\left[d_2^M (2{,}975.31; 3{,}000) \right] \right\}$$

$$= \frac{37.5}{1+1\% \times 6/12} \times \left\{ \Phi\left[\frac{\ln\left(3{,}000/2{,}975.31 \right) + \left(0.998\% - 2\% - (0.2)^2/2 \right) \times 0.5}{0.2 \times \sqrt{0.5}} \right] \right.$$

$$\left. - \left(\frac{2{,}975.31}{3{,}000} \right)^{\frac{2\times(-0.03002)}{0.2^2}} \Phi\left[\frac{\ln\left(2{,}975.31/3{,}000 \right) + \left(0.998\% - 2\% - (0.2)^2/2 \right) \times 0.5}{0.2 \times \sqrt{0.5}} \right] \right\}$$

$$= \frac{37.5}{1+1\% \times 6/12} \times \left\{ \Phi(-0.0477) - \left(\frac{2{,}975.31}{3{,}000} \right)^{\frac{2\times(-0.03002)}{0.2^2}} \Phi(-0.1646) \right\}$$

$\cong 1.53.$

Em suma,

$DI(1)_0 \left(S_0; X = 3{,}000; L = 2{,}975.31; R = 37.5; T = 0.5y \right)$

$= 138.30 + 1.53$

$\cong 139.83,$

e

$$RV_0 = \frac{40\%}{3{,}000} \times 139.83 \cong 1.864\%.$$

$B_0 = 99.50\% + 1.86\% = 101.36\% > 100\% \Rightarrow$ Depositar.

d)

$$B_0 = \frac{100\%}{1+1\% \times 6/12} + RV_0 .$$

$$RV_{6M} = 8\% \times 1_{\left\{ \inf_{0<u\le 0.5} (S_u) \le 3{,}000 \times 0.9 \right\}} .$$

Portanto,

$$RV_0 = 8\% \times e^{-0.998\% \times 0.5} \times Q\left[\inf_{0<u\le 0.5} (S_u) \le 2{,}700 \right]$$

$$= 8\% \times e^{-0.998\% \times 0.5} \times Q\left[\inf_{0<u\le 0.5} \left(\ln\left(\frac{S_u}{S_0} \right) \right) \le \ln\left(\frac{2{,}700}{S_0} \right) \right]$$

$$= 8\% \times e^{-0.998\% \times 0.5} \times \left\{ 1 - Q\left[\inf_{0<u\le 0.5} \left(\ln\left(\frac{S_u}{S_0} \right) \right) > \ln\left(\frac{2{,}700}{S_0} \right) \right] \right\}.$$

Via Proposição 45,

$$Q\left[\inf_{0<u\le 0.5} \left(\ln\left(\frac{S_u}{S_0} \right) \right) > \ln\left(\frac{2{,}700}{S_0} \right) \right]$$

$$= \Phi\left[d_2^M (3{,}000;2{,}700) \right] - \left(\frac{2{,}700}{3{,}000} \right)^{\frac{2\times(-0.03002)}{0.2^2}} \Phi\left[d_2^M (2{,}700;3{,}000) \right]$$

$$= \Phi\left[\frac{\ln\left(3{,}000/2{,}700 \right) + \left(0.998\% - 2\% - (0.2)^2/2 \right) \times 0.5}{0.2 \times \sqrt{0.5}} \right]$$

$$- \left(\frac{2{,}700}{3{,}000} \right)^{\frac{2\times(-0.03002)}{0.2^2}} \Phi\left[\frac{\ln\left(2{,}700/3{,}000 \right) + \left(0.998\% - 2\% - (0.2)^2/2 \right) \times 0.5}{0.2 \times \sqrt{0.5}} \right]$$

$$= \Phi(0.6389) - \left(\frac{2{,}700}{3{,}000} \right)^{\frac{2\times(-0.03002)}{0.2^2}} \Phi(-0.8512)$$

$$\cong 0.5074.$$

Portanto,

$$RV_0 = 8\% \times e^{-0.998\% \times 0.5} \times \{1 - 0.5074\}$$

$$\cong 3.92\%.$$

$B_0 = 99.50\% + 3.92\% = 103.42\% > 100\% \Rightarrow$ Depositar.